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Dissecting collinear splittings of quark and gluon jets at NNLL

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Zurich, 7th November 2023

Based mainly on work in JHEP 12 (2021) with Basem El-Menoufi and arXiv: 2307.15374 with Pier Monni, Basem El-Menoufi, Jack Helliwell and Melissa van Beekveld

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Outline

- Brief motivation for these studies
- Resummation coefficients at NNLL
- Beyond NLL in showers : Extending the K_{CMW} concept and derivation of $B_2\,(z)$
- Analysis of results : connection to known IRC safe observable results, fragmentation functions, effective emission probability
- NNLL resummed result for groomed jet observables
- Conclusions

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Motivation

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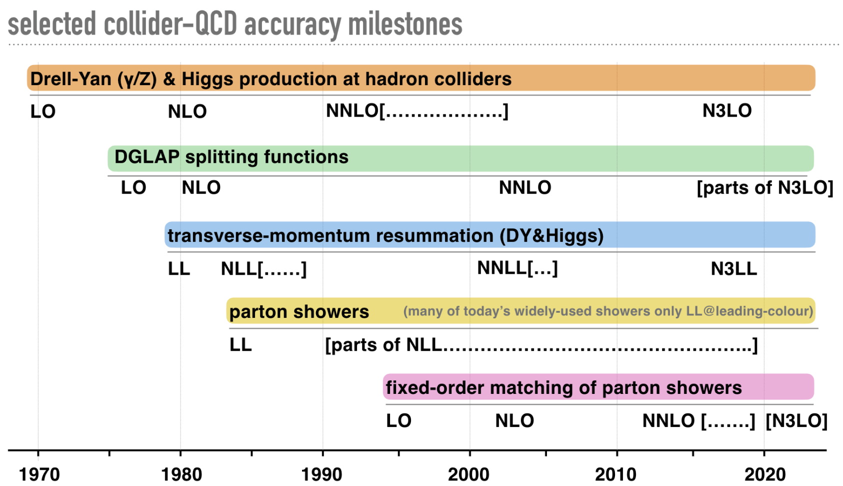
Parton shower accuracy



Taken from talk at Moriond QCD 2023 by G.Salam

- Log accuracy of showers under much scrutiny : a field that had stood relatively still for decades

- Over the same period substantial progress in understanding the structure of QCD in soft and collinear limits and in analytic resummation

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Logarithmic accuracy

$$\Sigma(Q) = \sum_n c_n \alpha_s^n$$

Single scale observable.
Accuracy specified by maximum n.

$$\Sigma(Q, vQ) = \sum_{n,m \leq 2n} c_{nm} \; \alpha_s^n L^m \qquad v \ll 1 \;\; L = \ln \frac{1}{v}$$

Multiscale observable.
Accuracy specified by n and m.

$$\Sigma(Q, vQ) \sim \exp[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots]$$

Multiscale observable **with exponentiation. Accuracy depends on g_n**

- g_1 is leading log (LL). Controls all double log (m= 2n) terms in expansion.
- Including g_2 gives NLL and g_3 is NNLL.
- **NLL is a must for accurate pheno**.

Catani, Trentadue, Turnock and Webber 1992

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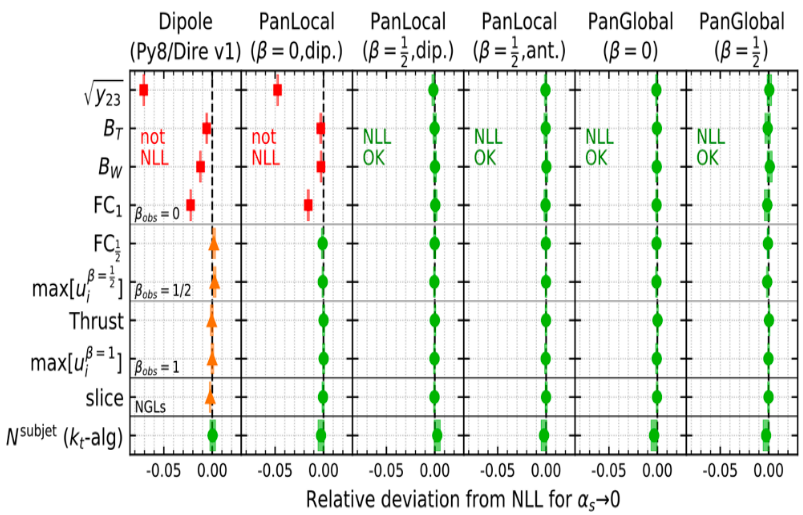
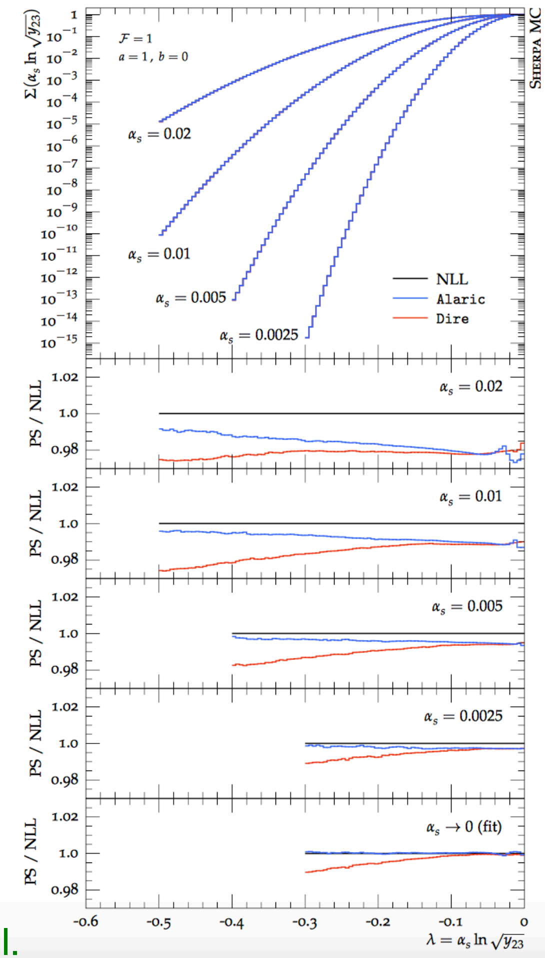
NLL accurate showers



MD, Dreyer, Monni, Hamilton, Salam & Soyez 2020.

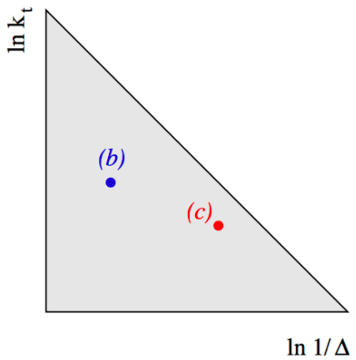


Herren, Hoeche, Schoenherr, Krauss 2022

- Several widely used dipole showers found to be only LL at leading Nc. MD et al .2018
- Principles identified for NLL MD et al. 2020
- Demonstrably NLL dipole showers constructed

MD et al 2020, Hamilton et al 2020, van Beekveld et al 2022, van Beekveld & Ferrario Ravasio 2023 , Nagy & Soper 2011, Forshaw et al. 2020, Herren et al. 2022

NLL criteria



$$\frac{d\mathcal{P}_{n\to n+1}}{d\ln v} = \sum_{\text{dipoles }\{\tilde{\imath},\tilde{\jmath}\}} \int d\bar{\eta} \frac{d\phi}{2\pi} \frac{\alpha_s(k_t) + K\alpha_s^2(k_t)}{\pi} \times \left[g(\bar{\eta}) a_k P_{\tilde{\imath}\to ik}(a_k) + g(-\bar{\eta}) b_k P_{\tilde{\jmath}\to jk}(b_k)\right],$$

Can we build on accuracy principles identified for NLL?

For NLL: MD, Dreyer, Hamilton, Monni, Salam, Soyez 2020

- Need to reproduce QCD matrix elements in limit where all emissions strongly ordered in at least one of 2 possible logarithmic variables
- Correct inclusion of virtual corrections. Here showers simply exploit unitarity. Only degree of freedom left is coupling scheme.

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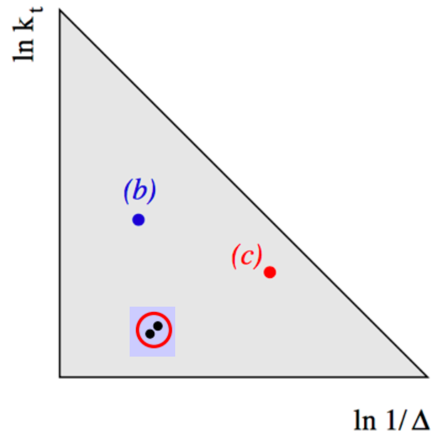
Towards NNLL?



This suggests need for

- Getting real emission matrix-elements right in limit where pair of emissions are close in Lund plane ⟹ higher-order splitting kernels
- Known for over 2 decades. Campbell and Glover 1997, Catani & Grazzini 1998
- Including suitable analytical ingredients to take care of virtuals. At NLL done via K. Catani, Marchesini Webber 1991. But beyond NLL we need more.

Ferrario Ravasio et al 2023.
First NNLL shower for soft limit

Hoeche, Krauss, Prestel 2017
Dulat, Hoeche, Prestel 2019

NNLL ingredients

$$S_c(Q,b) = \exp\left\{ -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[A_c(\alpha_S(q^2)) \ln\frac{Q^2}{q^2} + B_c(\alpha_S(q^2)) \right] \right\}$$

Collins Soper Sterman 1981

$$A_c(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)}$$

$$B_c(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)}$$

$$A_q^{(1)} = C_F \;, \quad A_q^{(2)} = \frac{1}{2} C_F K \quad B_q^{(1)} = -\frac{3}{2} C_F$$

Correctly taken care of in NLL

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{10}{9} T_R n_f$$

- Use NNLL resummation to guide us. Typified by form factor in CSS approach. But basic idea more general
- Resummation accuracy controlled by “A” series of coefficients for the soft limit and “B” series for hard-collinear limit
- To go to NNLL we **need to account in the collinear series for B_2** (and in soft series for A_3.)

B_2 and collinear emissions

- A_2 (or K_{CMW}) governs intensity of soft radiation from a hard parton. Related to a physical coupling definition in soft limit

- Similarly B_2 relates to intensity of collinear radiation off a given parton

- *Observable dependent* but always takes the form

$$B_2^f = -\gamma_f^{(2)} + b_0 X_v^f \qquad b_0 = \frac{11}{6} C_A - \frac{2}{3} T_R N_f$$

Davies and Stirling 1984
Catani, De Florian and Grazzini 2001
Banfi et al. 2019

$$\gamma_q^{(2)} = C_F^2 \left(\frac{3}{8} - \frac{\pi^2}{2} + 6\zeta(3) \right) + C_F C_A \left(\frac{17}{24} + \frac{11\pi^2}{18} - 3\zeta(3) \right) - C_F T_R n_f \left(\frac{1}{6} + \frac{2\pi^2}{9} \right)$$

$$-\gamma_g^{(2)} = \frac{4}{3} C_A T_R n_f + C_F T_R n_f - C_A^2 \left(\frac{8}{3} + 3\zeta_3 \right)$$

→ Endpoints of NLO DGLAP kernels

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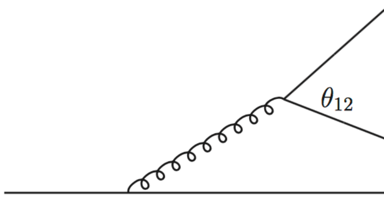
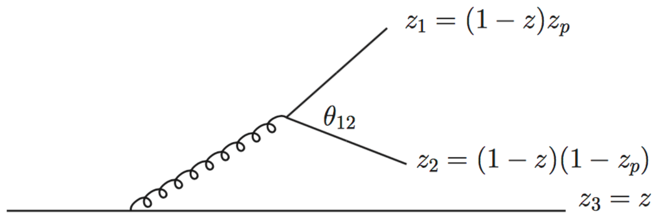
Computing a differential $B_2(z)$

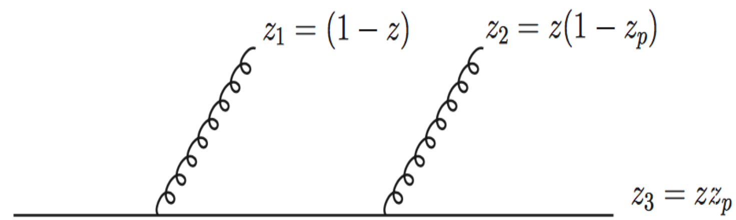


- In a shower approach we could encode info. on B_2 as function of emission kinematics
- Conceptually related to extension of K_{CMW} into collinear limit i.e. derive a function $B_2(z)$
- K_{CMW} computed from double-soft splitting kernels
- B_2 related to triple-collinear splittings

Splitting kernels : quarks

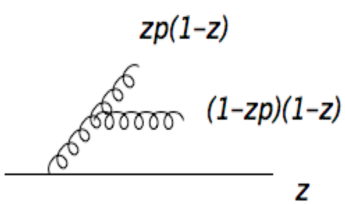


C_F T_R n_f and C_F $(C_F - C_A/2)$ pieces



Pure C_F^2 piece

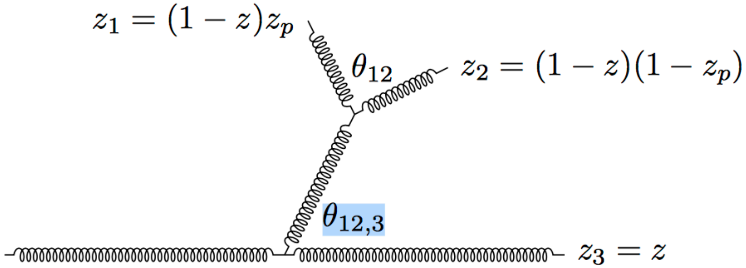
$$\left\langle \hat{P}_{\bar{q}_1' q_2' q_3} \right\rangle = \frac{1}{2} C_F T_R \frac{s_{123}}{s_{12}} \left[-\frac{t_{12,3}^2}{s_{12} s_{123}} + \frac{4z_3 + (z_1 - z_2)^2}{z_1 + z_2} + (1-2\varepsilon)\left(z_1 + z_2 - \frac{s_{12}}{s_{123}} \right) \right]$$



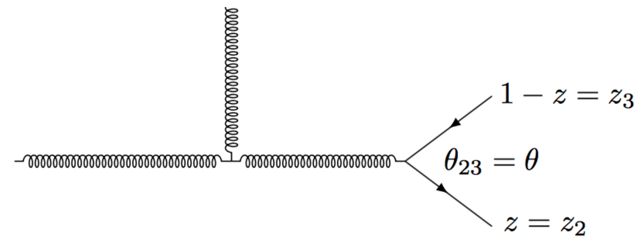
Pure C_F C_A piece

Quark jets have four distinct pieces from 3 branching processes.

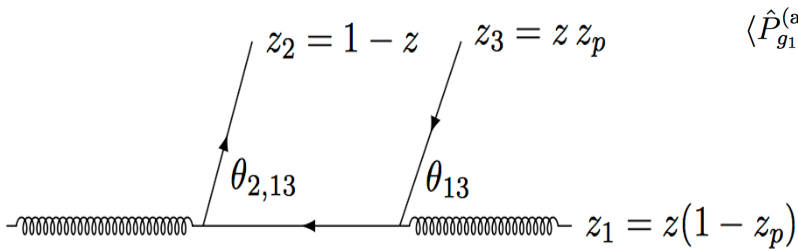
Splitting kernels : gluons



C_A^2 term



$C_A\, T_R\, n_f$ term

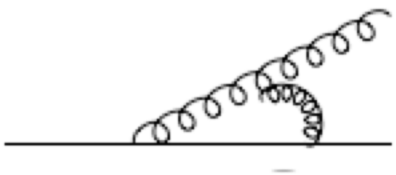


$$
\begin{aligned}
\langle \hat{P}^{(\mathrm{ab})}_{g_1 q_2 \bar{q}_3} \rangle &= -2 - (1-\epsilon)s_{23}\left(\frac{1}{s_{12}} + \frac{1}{s_{13}}\right) + 2\frac{s_{123}^2}{s_{12}s_{13}}\left(1 + z_1^2 - \frac{z_1 + 2z_2z_3}{1-\epsilon}\right) \\
&- \frac{s_{123}}{s_{12}}\left(1 + 2z_1 + \epsilon - 2\frac{z_1+z_2}{1-\epsilon}\right) - \frac{s_{123}}{s_{13}}\left(1 + 2z_1 + \epsilon - 2\frac{z_1+z_3}{1-\epsilon}\right)
\end{aligned}
$$

$C_F\, T_R\, n_F$ term

- 3 real emission kernels to consider
- Additionally a pure T_R^2 term from virtual corrections

Virtual corrections

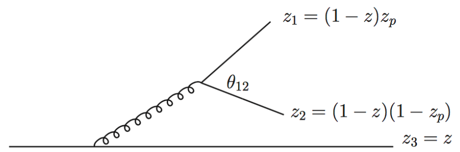


$$P^{(1)}_{q\to gq} = \frac{c_\Gamma g_s^2}{\epsilon^2}\left(\frac{-s_{12}-\imath 0}{\mu^2}\right)^{-\epsilon}\left[P^{(0)}_{q\to gq}\left(\frac{(C_F-C_A)\left(\epsilon(\delta\epsilon^2+\epsilon-3)+1\right)}{(\epsilon-1)(2\epsilon-1)}\right.\right.$$
$$+\left(C_A-2C_F\right)\,{}_2F_1\left(1,-\epsilon;1-\epsilon;\frac{z_1}{z_1-1}\right)-C_A\,{}_2F_1\left(1,-\epsilon;1-\epsilon;\frac{z_1-1}{z_1}\right)+C_F\Bigg)$$
$$\left.+\frac{g_s^2 C_F}{z_1}\,\frac{(z_1-2)(z_1-1)\epsilon^2(\delta\epsilon-1)\left(C_A-C_F\right)}{(\epsilon-1)(2\epsilon-1)}\right]+\text{c.c.}\,,$$

- Also need the one-loop corrections to a collinear 1 to 2 splitting
- Taken from De Florian, Rodrigo, Sborlini (2013)
- Perform an integral over real emission phase space at fixed kinematics for a suitably defined first splitting. Do this in dim. reg. and combine with virtual piece.

Calculations and Results

Calculations : set up



- We compute a collinear limit NLO correction to a given 1 to 2 splitting

$$\frac{\theta^2}{\sigma_0}\frac{d^2\sigma}{dz\,d\theta^2} = \int \mathrm{d}\Phi_3(z_i,\theta_{ij})\frac{\left(8\pi\alpha_s\mu^{2\epsilon}\right)^2}{s_{123}^2}\langle\hat{P}\rangle\,\theta^2\,\delta\left(\theta^2-\theta^2\left(z_i,\theta_{ij}\right)\right)\delta(z-z(z_i))\,\Theta_{\mathrm{cut}}(\theta_{ij})$$

- Fix energy and angle of the initial collinear splitting
- Our definition of energy fraction and angle are based on triple collinear configurations.
- Multiple definitions possible but in soft and collinear limits always point back to a unique 1 to 2 splitting.

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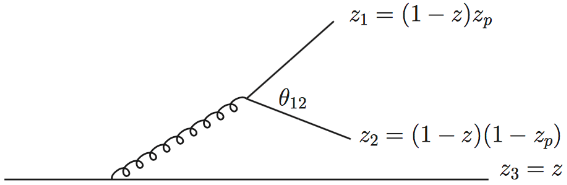
Results : $C_F\ T_R\ n_f$ piece



Consider fixing z and parent angle $\theta_g^2 = z_p\theta_{13}^2 + (1-z_p)\theta_{23}^2 - z_p(1-z_p)\theta_{12}^2$

or other related quantity e.g. jet mass $\rho = z_1 z_3\theta_{13}^2 + z_2 z_3\theta_{23}^2 + z_1 z_2\theta_{12}^2$

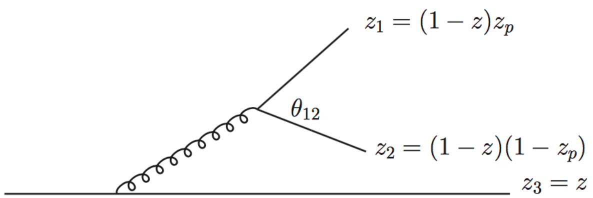
$$\left(\frac{\rho}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\rho\, dz}\right)^{C_F T_R n_f} = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\left(\frac{2}{3}\ln\left(\rho(1-z)\right) - \frac{10}{9}\right) - \frac{2}{3}(1-z)\right)$$

$$\left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta_g^2\, dz}\right)^{C_F T_R n_f} = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\left(\frac{2}{3}\ln\left(z(1-z)^2\theta_g^2\right) - \frac{10}{9}\right) - \frac{2}{3}(1-z)\right)$$

Also recovers soft limit exp. for scale and scheme

- NLO Results related by LO substitution
- Effect of gluon virtuality incorporated in K and z dependence
- Results contain info. on scale and scheme of coupling beyond soft limit

Results : $C_F(C_F-C_A/2)$ piece



- Identical particle interference term is purely finite.. Cannot identify parent uniquely but this ambiguity is irrelevant. Fixing any angle gives same result

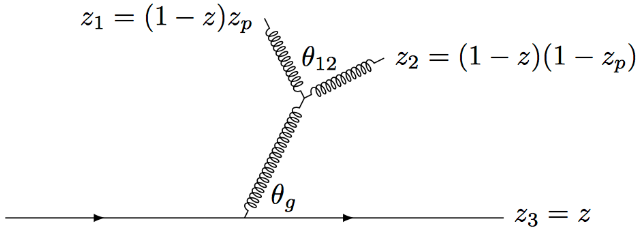
- Can look at either quark or antiquark distribution. Fixing z of either quark gives :

$$\left(\frac{\rho}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\rho\, dz}\right)^{(\mathrm{id.})} = \left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta_g^2\, dz}\right)^{(\mathrm{id.})} = C_F\left(C_F - \frac{C_A}{2}\right)\left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{P}^{(\mathrm{id.})}(z),$$

$$\mathcal{P}^{(\mathrm{id.})}(z) = \left(4z - \frac{7}{2}\right) + \frac{5z^2-2}{2(1-z)}\ln z + \frac{1+z^2}{1-z}\left(\frac{\pi^2}{6} - \ln z \ln(1-z) - \mathrm{Li}_2(z)\right).$$

- Fixing z of anti-quark gives directly the non-singlet MSbar fragmentaton function $P_{q\bar{q}}^{V,(1)}$ (Eq. 4.108 of Ellis, Stirling, Webber text)

Results : pure C_F C_A piece



More involved calc. due to soft divergences.
Final answer similar to nf piece.

$$\left(\frac{\rho}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\rho\, dz}\right)^{\text{nab.}} = C_F C_A \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{P}^{(\text{nab.})}(z;\rho)$$

$$\mathcal{P}^{(\text{nab.})}(z;\rho) = \left(\frac{1+z^2}{1-z}\right)\left(-\frac{11}{6}\ln\left(\rho(1-z)\right) + \frac{67}{18} - \frac{\pi^2}{6} + \ln^2 z + \mathrm{Li}_2\left(\frac{z-1}{z}\right) + 2\,\mathrm{Li}_2(1-z)\right) + $$
$$+\frac{3}{2}\frac{z^2\ln z}{1-z} + \frac{1}{6}(8-5z)$$

Note again appearance of KCMW coeff. and b_0 ln k_t term with rest giving hard collinear extension

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Extracting $B_2(z)$

- Involves removing higher log order ingredients from our results.
- Illustrate on n_f term for quark jets

$$\left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta_g^2\,dz}\right)^{C_F T_R n_f} = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\left(\frac{2}{3}\ln\left(z(1-z)^2\theta_g^2\right) - \frac{10}{9}\right) - \frac{2}{3}(1-z)\right)$$

First remove soft limit terms

$$\left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta_g^2\,dz}\right)^{\text{soft},C_F T_R n_f} = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{2}{1-z}\left(\frac{2}{3}\ln\left((1-z)^2\theta_g^2\right) - \frac{10}{9}\right)$$

$$= C_F \frac{2}{1-z}\left(\frac{\alpha_s}{2\pi}\right)^2\left(-b_0^{(n_f)}\ln\frac{k_t^2}{E^2} + K^{(n_f)}\right) ,$$

Remove also remaining NLL hard collinear term $\propto -(1+z)\ln\theta_g^2$

$$\mathcal{B}_2^{q,n_f}(z;\theta_g^2) = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\frac{2}{3}\ln z - (1+z)\left(\frac{2}{3}\ln(1-z)^2 - \frac{10}{9}\right) - \frac{2}{3}(1-z)\right)$$

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Extracting $B_2(z)$: gluon splitting channels

Similar exercise gives pure C_F CA term :

$$\mathcal{B}_2^{q,(\text{nab.})}(z;\theta_g^2) = C_F C_A \left(\frac{\alpha_s}{2\pi}\right)^2 \left((1+z)\left(\frac{11}{6}\ln(1-z)^2 - \frac{67}{18} + \frac{\pi^2}{6}\right) + \frac{3}{2}\frac{z^2 \ln z}{1-z} + \frac{8-5z}{6} + \frac{1+z^2}{1-z}\left(-\frac{11}{6}\ln z + \ln^2 z + \text{Li}_2\left(\frac{z-1}{z}\right) + 2\text{Li}_2(1-z)\right)\right)$$

Results for other variables follow from single emission kinematic relationship e.g.

$$\mathcal{B}_2^{q,n_f}(z;\rho) = \mathcal{B}_2^{q,n_f}(z;\theta_g^2) - C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\frac{2}{3}\ln z - (1+z)\frac{2}{3}\ln(1-z)\right)$$

For analogous C_F C_A relation replace 2/3 by -11/6.

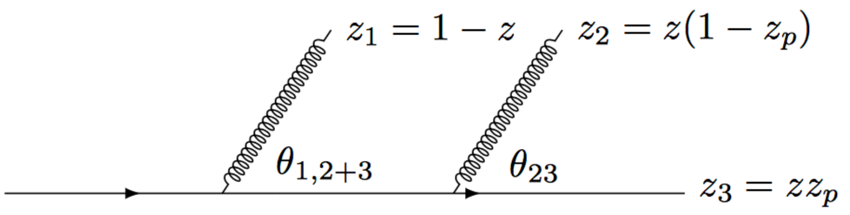
Identical fermion term universal (no NLL piece to remove):

$$\mathcal{B}_2^{q,(\text{id.})}(z) = C_F\left(C_F - \frac{C_A}{2}\right)\left(\frac{\alpha_s}{2\pi}\right)^2 \left(4z - \frac{7}{2}\right) + \frac{5z^2-2}{2(1-z)}\ln z + \frac{1+z^2}{1-z}\left(\frac{\pi^2}{6} - \ln z \ln(1-z) - \text{Li}_2(z)\right)$$

$B_2(z)$ for $C_F{}^2$ channel

- Here definition of "first emission" may be given by some ordering

- We take ordering in angle as simple choice with $\theta_{13} > \theta_{23}$

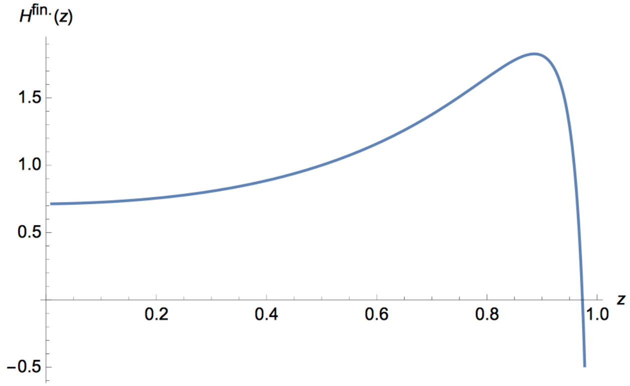


- Obtain B_2 as difference between triple-collinear and iterated 1 to 2 splittings and phase-space + virtual corr. to 1 to 2 splitting.

- Schematically amounts to computing

$$B_2(z) = \int d\Phi_3\, P_{1\to 3}\, \delta(z - (1 - z_1))\, \theta(\theta_{13} > \theta_{23}) - \int d\Phi_2^2\, P_{1\to 2}^2\, \delta(z - (1 - z_1))\, \theta(\theta_{13} > \theta_{23}) + V_1(z, \epsilon)$$

$B_2(z)$ for $C_F{}^2$ channel



Here due to ordering part of the result is numerical :

$$\mathcal{B}_2^{q,(\text{ab.})}(z) = \left(\frac{C_F\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\left(-3\ln z + 2\text{Li}_2\left(\frac{z-1}{z}\right) - 2\ln z\ln(1-z)\right) - 1 + H^{\text{fin.}}(z)\right)$$

$$\int_0^1 H^{\text{fin}}(z)\,dz = 4\zeta(3) - \frac{31}{8}$$

Integrals over z

Integrals over z produce the expected form

$$B_2^{q,(\mathrm{ab.})} = \left(\frac{2\pi}{\alpha_s}\right)^2 \int_0^1 \mathcal{B}_2^{q,\mathrm{ab.}}(z)\, dz = \pi^2 - 8\zeta(3) - \frac{29}{8}$$

$$B_2^{q,(\mathrm{ab.})} + B_2^{q,(\mathrm{id.}),C_F^2} = -\gamma_q^{(2,C_F^2)} = C_F^2\left(\frac{\pi^2}{2} - 6\zeta(3) - \frac{3}{8}\right)$$

Combining also other channels we recover full $-\gamma_q^{(2} + C_F b_0 X_v$

with

$$X_{\theta_g^2} = \frac{2\pi^2}{3} - \frac{13}{2}$$
$$X_\rho = \frac{\pi^2}{3} - \frac{7}{2}$$

Agrees with hard-collinear NNLL piece in resummation literature

Becher & Schwartz 2008. Banfi et. al. 2014, 2019

Connecting to fragmentation functions

- Natural to expect link between this work and NLO DGLAP kernels for non-singlet time-like splittings

- Direct link for those pieces where z has the same meaning . Looking at NNLL structure of results for n_f piece we can write it as

$$P^{\mathrm{NLO},n_f}(z;\theta_g^2) = C_F T_R n_f \left[\frac{1+z^2}{1-z}\left(-\frac{2}{3}\ln z - \frac{10}{9}\right) - \frac{4}{3}(1-z)\right] + \\ + C_F T_R n_f \left[\frac{1+z^2}{1-z}\left(\frac{2}{3}\ln(1-z)^2 + \frac{2}{3}\ln z^2\right) + \frac{2}{3}(1-z)\right]$$

- Bottom line gives b_0 X term on integration. Top line is NLO time-like non-singlet splitting function $P_{qq}^{V(1),n_f}$

Connection to fragmentation functions

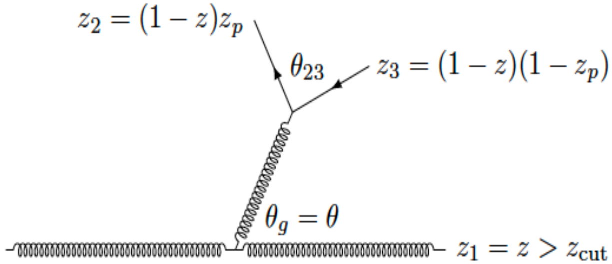
- Similarly from pure C_F C_A piece remove ''b_0 X" terms (related to n_f ones by 2/3 ⇨ -11/6).

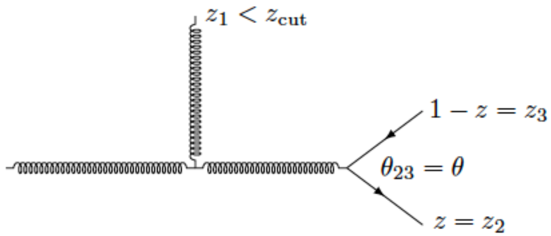
- Then add twice our result from $C_F(C_F - C_A/2)$ identical fermion term

$$P_{\text{sub.}}^{\text{NLO,nab.}}(z,\rho) + 2 \times \left(-C_F \frac{C_A}{2}\right) \mathcal{P}^{(\text{id.})}(z) = C_F C_A \left[\frac{1+z^2}{1-z}\left(\frac{1}{2}\ln^2 z + \frac{11}{6}\ln z + \frac{67}{18} - \frac{\pi^2}{6}\right) + (1+z)\ln z + \frac{20}{3}(1-z)\right] ,$$

RHS is NLO DGLAP result for $P_{qq}^{V(1),C_F C_A}$

Gluon jet subtleties





- Gluon jets can be handled similarly modulo a few mainly technical differences/subtleties.
- Two histories involved at Born level
- Independent and correlated emission pictures mixed within same colour channel
- IR divergences associated to both emissions in a g to gg branching. Makes definition of basic 1 to 2 splitting more subtle.

IRC safe procedure for defining z and θ based on SoftDrop declustering

arXiv:2307.15
7342

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Gluon jets results summary

- We obtain results for each channel with a fully analytic result in the $C_A\, T_R\, n_f$

$$\mathcal{B}_2^{g,C_A T_R}(z) = -p_{qg}(z)\left(\ln^2 z + \ln^2(1-z)\right) + \frac{1}{9}(28 - 41z + 41z^2) + \ln z\left(\frac{4}{3(1-z)} - \frac{26}{3}z^2 + 8z - 7\right) + \ln(1-z)\left(\frac{4}{3z} - \frac{26}{3}(1-z)^2 + 8(1-z) - 7\right)$$

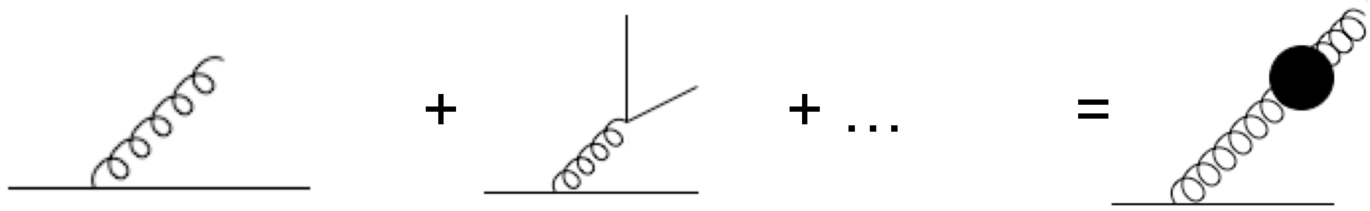
- In other channels we have a semi-analytic result c.f. the C_F^2 channel of quark jets The results are consistent with

$$\int_0^1 dz\, B_2(z) = -\gamma_g^{(2)} + b_0 X_\theta^2 \quad X_\theta^2 = \left(-\frac{67}{9} + \frac{2\pi^2}{3}\right) C_A + \frac{23}{9} T_R n_f$$

N.B. In the C_A^2 channel a clustering correction appears specific to C/A clustering. Identical to that of SoftDrop jet mass.

Applications

Shower emission probability



- Look to define an effective emission prob. relevant for NNLL Sudakov in parton showers or jet calculus.

- Consider combination with LO . Can express as

$$\left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma}{d\theta_g^2 dz}\right)^{\text{tot.}} = C_F\left(\frac{1+z^2}{1-z}\right)\left[\frac{\alpha_s\left(E^2\right)}{2\pi} + \left(\frac{\alpha_s}{2\pi}\right)^2\left(-b_0\ln\left((1-z)^2\theta_g^2\right)+K\right) - \left(\frac{\alpha_s}{2\pi}\right)^2 b_0 \ln z\right] + \mathcal{R}$$

Suggests modification of argument of running coupling in h.c. limit

$$\equiv \frac{C_F}{2\pi}\left(\frac{1+z^2}{1-z}\right)\alpha_s\left(E_j^2 z(1-z)^2\theta_g^2\right)\left(1+\frac{\alpha_s}{2\pi}\mathcal{K}(z)\right)$$

Emission probability and Sudakov

More explicitly emission probability (e.g. for quark jets) reads

$$\mathcal{P}_q(z,\theta) \equiv \frac{2\,C_F}{1-z}\left(1+\frac{\alpha_s(E^2g^2(z)\theta^2)}{2\pi}K^{(1)}\right) + \mathcal{B}_1^q(z) + \frac{\alpha_s(E^2g^2(z)\theta^2)}{2\pi}\left(\mathcal{B}_2^q(z)+\mathcal{B}_1^q(z)b_0\ln g^2(z)\right)$$

Related to this we have a Sudakov form factor

$$\ln\Delta_q(t) = -\int_t^{t_0} dt' \int_{z_0}^{1-z_0} dz\,\mathcal{P}_q(z,\theta)$$

These would form 2 of the main elements of a higher order shower.

In the most general case we would then need the 1 to 3 corrections in the shower

NNLL for groomed jet observables

- Effective emission prob. defines NNLL Sudakov correct in h.c. limit
- Need also soft limit ingredients for full story
- However already possible to directly exploit for pure collinear observables.
- Insight led to new NNLL resummed results for groomed variables measured at the LHC.

NNLL for groomed observables

- Considered groomed angularities and groomed fractional moments of EEC

$$FC_x^{\mathcal{H}} = \frac{2^{-x}}{E^2} \sum_{i \neq j} E_i E_j |\sin\theta_{ij}|^x (1 - |\cos\theta_{ij}|)^{1-x}$$

$$\lambda_x^{\mathcal{H}} = \frac{2^{1-x}}{E} \sum_i E_i |\sin\theta_i|^x (1 - |\cos\theta_i|)^{1-x},$$

Sum runs over particles in hemisphere

Then define

MD et al 2023

$$FC_x \equiv \max\left\{FC_x^{\mathcal{H}_R}, FC_x^{\mathcal{H}_L}\right\}, \; \lambda_x \equiv \max\left\{\lambda_x^{\mathcal{H}_R}, \lambda_x^{\mathcal{H}_L}\right\}$$

NNLL from emission probability

- Results take the form

$$\Sigma^q(v) = \sigma_0^{Z\to q\bar{q}}\left(1+\frac{\alpha_s(E^2)}{2\pi}C_v^{q(1)}(z_{\rm cut})\right)e^{-2R_v^q(v,z_{\rm cut})}\left(1+\frac{\alpha_s^2(E^2)}{(2\pi)^2}\,2\mathcal{F}^q_{\rm clust}(v)\right)$$

$$\Sigma^g(v) = \sigma_0^{H\to gg}\left(1+\frac{\alpha_s(E^2)}{2\pi}C_v^{g(1)}(z_{\rm cut})\right)e^{-2R_v^g(v,z_{\rm cut})}\left(1+\frac{\alpha_s^2(E^2)}{(2\pi)^2}\,2\mathcal{F}^g_{\rm clust}(v)\right)$$

The NNLL piece of the Sudakov comes from the integral of emission probability for quark and gluon jets e.g. for angularities:

$$B^q_{2,\lambda_x} = B^q_{2,\theta^2} + C_F\, b_0\frac{(9-\pi^2+9\ln 2)}{3(2-x)} = -\gamma_q^{(2)} + b_0 X^q_{\lambda_x}\,,$$

$$B^g_{2,\lambda_x} = B^g_{2,\theta^2} + \frac{b_0}{2-x}\left(C_A\left(\frac{137}{36}-\frac{\pi^2}{3}+\frac{44\ln 2}{12}\right)-T_R n_f\left(\frac{29}{18}+\frac{4\ln 2}{3}\right)\right)$$

Results extend to jets in any hard process e.g. LHC processes

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Summary and Conclusions

- NLL showers becoming well established. NNLL accuracy becomes realistic next target
- Needs higher-order kernels + specific analytic ingredients
- Discussed one key ingredient $B_2(z)$. This gives NNLL hard collinear Sudakov form factor
- Recovered standard results in hard-collinear limit for IRC safe observables and derive new results for groomed obervables plus establised contact with NLO DGLAP splitting kernels
- Much work to be done in terms of inclusion in parton showers.