week 5 quiz is online

PHY117 HS2023

Week 6, Lecture 2
Oct. 25th, 2023
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The ideal gas law is an approximation, assuming the molecules are point particles (no size), and it neglects the force between the molecules.

$$PV = nRT$$

More accurate:

$$\boxed{\left(P + \frac{an^2}{V^2}\right)(V - bn) = nRT}$$

Van der Waals equation

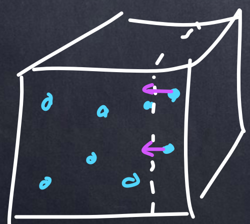
correction to the volume

b: volume of 1 mole of molecules

bn: Volume of n moles of molecules

a: constant that depends on the attractive molecular forces.

there is a decrease in pressure against the walls of our container due to molecular forces.

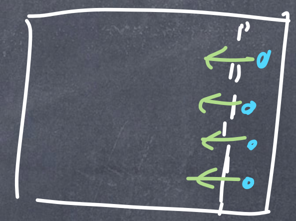


each molecule near the right wall feels a net attractive force from the molecules to the left.

The force pulling on one molecule near the wall is proportional to the density on the molecules $\sim \frac{n}{V}$

The number of molecules near the wall is also proportional to the density of molecules $\sim \frac{n}{V}$

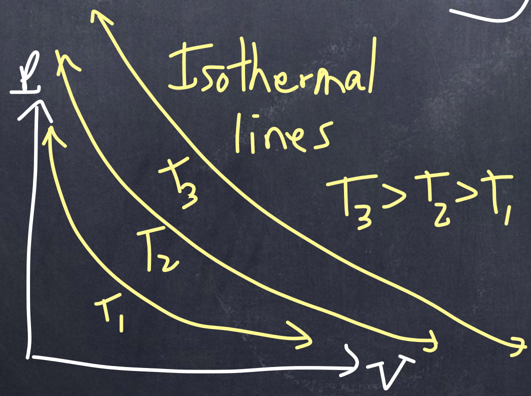
So the total force is $\sim \frac{n^2}{V^2}$



Note: Is the pressure more or less due to the attractive forces (a)

$$P = \frac{nRT}{V-bn} - \underbrace{\frac{an^2}{V^2}}$$ pressure decreases

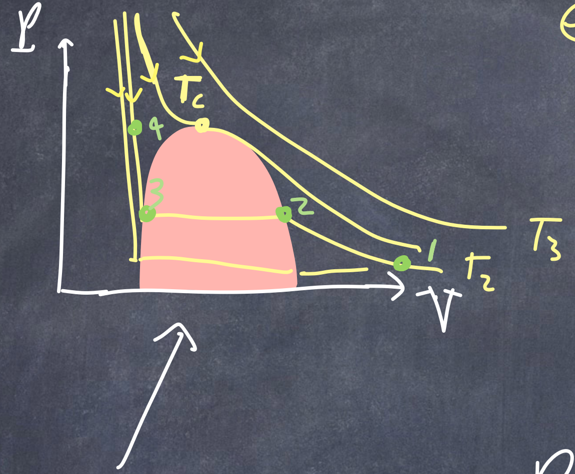
Ideal gas law



The ideal gas law is only valid above a certain temperature, T_c : critical temperature.

Above T_c, the behavior of the gas is described by the Van der Waals equation. But below T_c, we see something different.

Each curve is for a constant temperature



Starting at low pressure & high volume at ①, we compress the gas. The pressure rises to ② as expected from the gas law.

But between ② and ③, pressure stops rising and between ③ & ④, it increases sharply. Why?

② → ③ : gas begins to liquefy (both gas & liquid exist)

③ → ④ : only liquid exists. Since liquid is nearly incompressible, pressure increases rapidly while volume changes little.

Relationship between temperature and heat

Heat : is a form of energy.

we can add heat or remove heat.

symbol, Q.

No work done $\rightarrow$ $Q = mc\Delta T$

Q: heat added [J]

m: mass of a substance [kg]

c : specific heat of a substance

ΔT : temperature change

$\Delta T = T_f - T_i$ [K]

$$c : \left[\frac{J}{kg \cdot K}\right]$$

In a few slides, we find: $Q = \Delta U + W$.

Here, if no work done $Q = \Delta U = mc\Delta T$.

substances	$c\ \left[\frac{J}{kg \cdot K}\right]$	$C_m\ \left[\frac{J}{mol \cdot K}\right]$
copper	386	24.5
aluminum	900	24.2
silicon	710	42.2
water	$\boxed{4186}$	75.3
pine wood	1500	
oak wood	2400	

$$Q = mc\,\Delta T$$

$$Q = nC_m\,\Delta T$$

→ should have biggest temp. change in our experiment.

This means water is good at storing heat energy, and only changes temperature slightly.

forests are also good at moderating temperature

A big lake moderates temperature changes nearby.
Keeps summers cooler + winters warmer

Since heat is a form of energy,
we can use heat to do work.

1st law of thermodynamics:

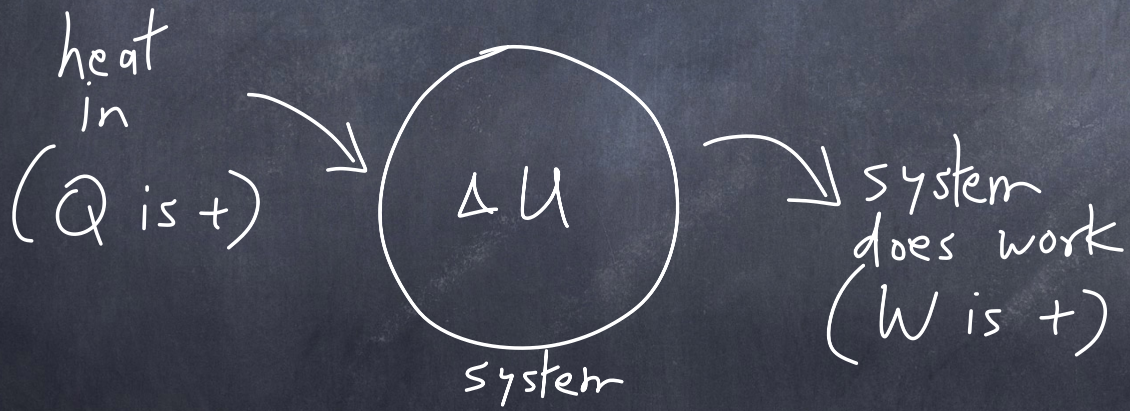
a statement of energy conservation:

$$Q = \Delta U + W$$

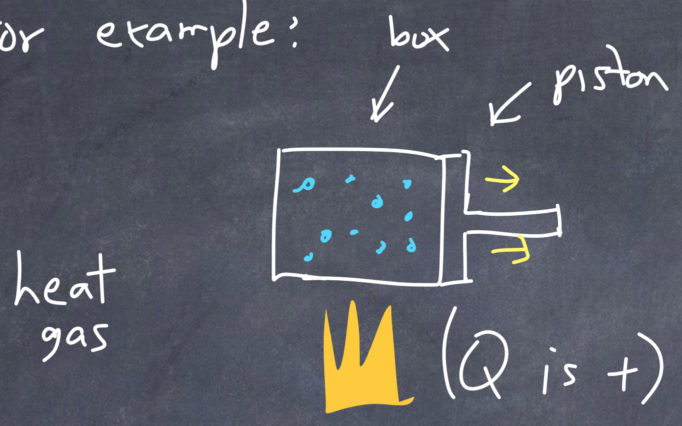
Q: heat <u>added</u> to a system

ΔU: change in internal energy of the system

W: work done <u>by</u> the system.



For example:



This causes the piston to move out because the gas expands.

The work is +, (the work done by the system.)

ΔU would increase (+)

Example: 3 kg of water at 80°C. We stir it with 15 kJ of energy. we also remove 50 kJ of heat. What is the final temperature?

$$Q = \Delta U + W$$

Here: $Q = -50$ kJ   Heat removed so (−)

$W = -15$ kJ   work done on system so (−)

$$\Delta U = Q - W = -50\text{ kJ} - (-15\text{ kJ})$$

$$= -35\text{ kJ}$$

The change in internal energy is (−).
The temperature will decrease.

For water, the volume & pressure are not changing.

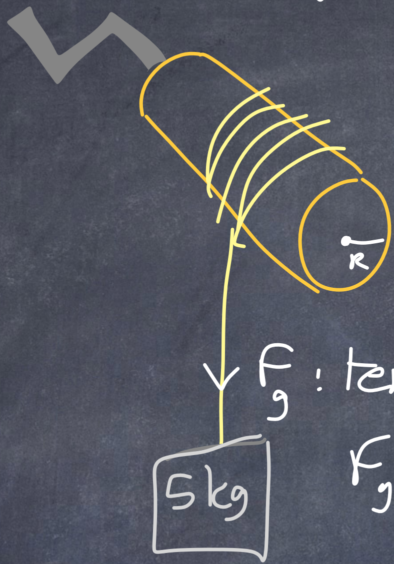
$$\Delta U = mc\Delta T$$

$$\Delta T = \frac{\Delta U}{mc} = \frac{-35\text{E}3\ \cancel{\text{J}}}{(3\ \cancel{\text{kg}})\left(4184\ \frac{\cancel{\text{J}}}{\cancel{\text{kg}}\cdot \text{K}}\right)} = -2.8\ \text{K}$$

$$= -2.8\,^{\circ}\text{C}$$

ΔT is the same in K & °C, but not T!

Can we use torque to increase temperature?



F_g : tension string

$F_g = Mg$

$F_f = F_g$

$\bar{\tau} = \bar{r} \times \bar{F} = RMg$

The work done on the cylinder to rotate it N times

$$W = \int_0^{2\pi N} \tau \, d\theta$$

$$W = \int_0^{2\pi N} RMg \, d\theta = MgR\theta \Big]_0^{2\pi N} = MgR\,2\pi N$$

work done on the system.

$$Q = \Delta U + W$$

Q : no heat added $= 0$

$$\Delta U = -W = -(-MgR\,2\pi N) = +MgR\,2\pi N$$

ΔU is the increase in internal energy of water + copper

$$(mc\Delta T)_{water} + (mc\Delta T)_{copper} = MgR\,2\pi N$$

$$\Delta U = (mc\Delta T)_{water} + (mc\Delta T)_{copper}$$

at equilibrium, $\Delta T_{water} = \Delta T_{copper}$

$C_w : 4186 \frac{J}{kg \cdot K}$ $\quad$ $C_c : 386\ J/kg \cdot K$

$$\Delta U \simeq \Delta T\,(m_w C_w + \cancel{m_c C_c}^{\,0}) = MgR\,2\pi N$$

small

$$\Delta T = \frac{2\pi N M g R}{m_w C_w}$$

$N = 100$
$R = 0.023\,m$
$m_w = 0.05\,kg$
$M = 5\,kg$

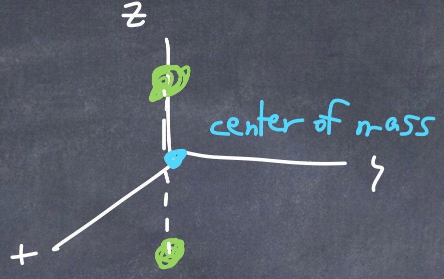
$\Delta T = 3\,k = 3^{\circ}C$

prediction if W becomes internal energy that increases water temperature

Remember! a molecule has $\frac{1}{2}kT$ of kinetic energy per degree of freedom.
(or $\frac{1}{2}RT$ per mole)

Equipartition theorem: when a substance is in equilibrium, there is an average energy of $\frac{1}{2}kT$ per molecule or $\frac{1}{2}RT$ per mole associated with each degree of freedom. The total is called the internal energy, U.

Consider a diatomic molecule in a gas $(N_2, O_2, N_2 \ldots)$
(at constant volume)



It can rotate around the x-axis or the y-axis so it has rotational kinetic energy.

$$K_{rot} = \frac{1}{2} I_x \omega_x^2 + \frac{1}{2} I_y \omega_y^2$$

The total kinetic energy is then:

for 1 molecule

$$K = \underbrace{\frac{1}{2} m v_x^2}_{\frac{1}{2}kT} + \underbrace{\frac{1}{2} m v_y^2}_{\frac{1}{2}kT} + \underbrace{\frac{1}{2} m v_z^2}_{\frac{1}{2}kT} + \underbrace{\frac{1}{2} I_x \omega_x^2}_{\frac{1}{2}kT} + \underbrace{\frac{1}{2} I_y \omega_y^2}_{\frac{1}{2}kT}$$

For N molecules, with 5 degrees of freedom,

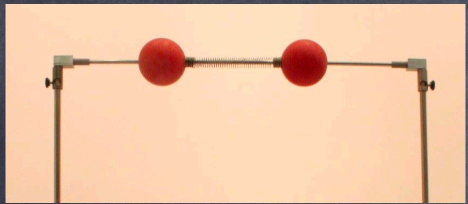
$$U = 5N\left(\frac{1}{2}kT\right) = \frac{5}{2}NkT = \frac{5}{2}nRT$$

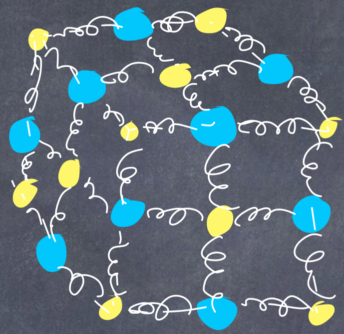
Note: if we increase U, we increase T

$$\Delta U = \frac{5}{2} nR\Delta T \Rightarrow \frac{\Delta U}{\Delta T} = \frac{5}{2}nR = c_V$$

heat capacity of diatomic gas at constant volume.

Likewise, for a solid, such as NaCl





Atoms are held together bound like springs.

$$K = \underbrace{\tfrac{1}{2}mv_x^2 + \tfrac{1}{2}mv_y^2 + \tfrac{1}{2}mv_z^2}_{\text{translational}} + \underbrace{\tfrac{1}{2}k_s x^2 + \tfrac{1}{2}k_s y^2 + \tfrac{1}{2}k_s z^2}_{\text{springs in 3-D}}$$

For 6 degrees of freedom:

$$U = 6 \cdot \tfrac{1}{2} nRT = 3nRT$$

$$U = 6 \cdot \tfrac{1}{2} NkT = 3NkT$$

Note: if we increase U, T increases:

$$\Delta U = 3nR\Delta T$$

So $\frac{\Delta U}{\Delta T} = 3nR$ for solids with 6 d.o.f.

$$\frac{\Delta U}{\Delta T} = 3R \text{ per mole}$$

$$= 24.9 \ J/mol \cdot K$$

This is very close to the value of C_m from our previous table

$$C_m \cong 3R \quad \text{and} \quad \frac{\Delta U}{\Delta T} = n\,C_m$$

Be careful with units on C

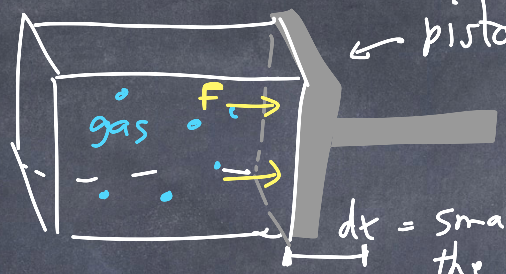
C can be given in several units:

$$\left[\frac{J}{mol \cdot K}\right] \quad \left[\frac{J}{kg \cdot K}\right] \quad \left[\frac{kcal}{kg \cdot {}^\circ C}\right]$$

check $U : [U]$

$T : [K]$

work done by a gas to move a piston



$\leftarrow$ piston has area, A

dx = small distance the piston moves

$F = PA$

$$A\Delta x = \Delta V$$
$$\downarrow$$
$$A\,dx = dV$$

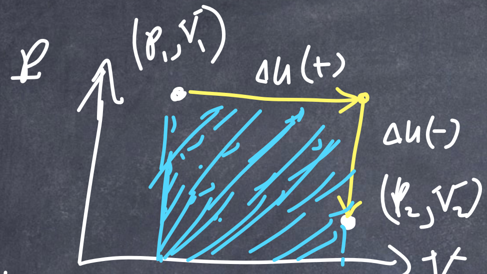
$$dW = F\,dx = PA\,dx = P\,dV$$

$$\int dW = \int P\,dV \Rightarrow W = \int P\,dV$$

$$\boxed{W = \int_{V_1}^{V_2} P\,dV}$$

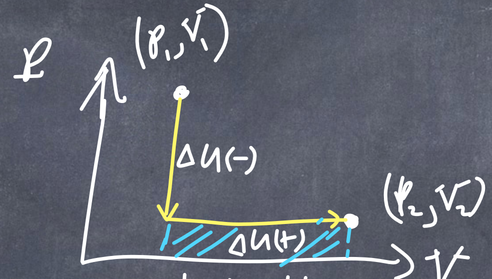
work done by a gas is the area under a P vs. V curve.

To go from (P_1, V_1) to (P_2, V_2) it depends on how we do it.



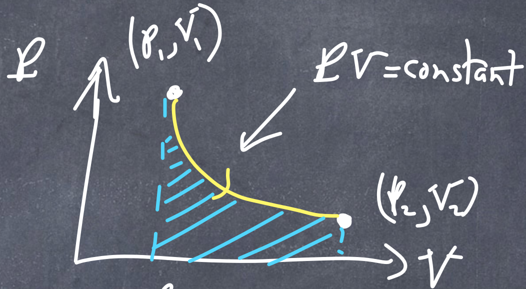
heat gas allowing it to expand then fix the volume, and cool the gas.

$$W = P_1(V_2 - V_1)$$



cooled the gas at constant volume, then heated gas at constant pressure

$$W = P_2(V_2 - V_1)$$



$PV = nRT$

no $\Delta T \Rightarrow$ no $\Delta U = 0$

Heat the gas

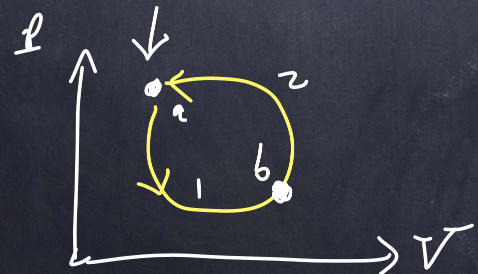
$$W = \int_{V_1}^{V_2} P\,dV$$

$$P = \frac{nRT}{V} \Rightarrow W = nRT\int_{V_1}^{V_2} \frac{dV}{V}$$

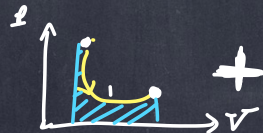
$$\boxed{W = nRT \ln \frac{V_2}{V_1}}$$

cycle: $\Delta U = 0$, net work

$a \to a$



$$= \underbrace{\quad (W>0) \quad + \quad (W<0) \quad}_{\text{work}}$$



$W > 0$

$V_2 > V_1$

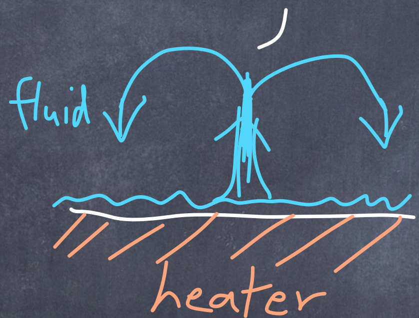
+ work, expands

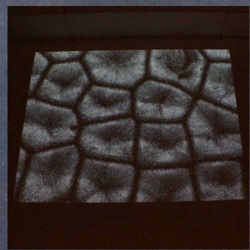
$W < 0$

$V_2 < V_1$

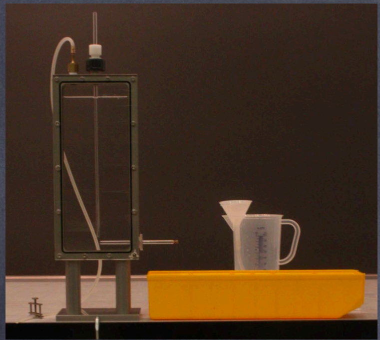
Transfer of thermal energy is done by 3 main processes: conduction
convection
radiation

convection: heat transported by a mass of material moving. For instance, hot air is less dense and it rises.





radiation: energy absorbed + emitted in electromagnetic radiation (visible light, infrared light, X-rays)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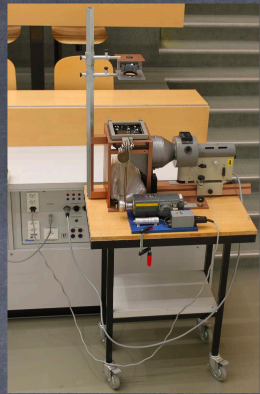
Th57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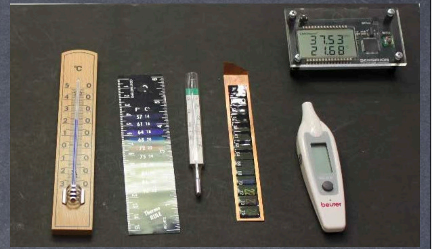
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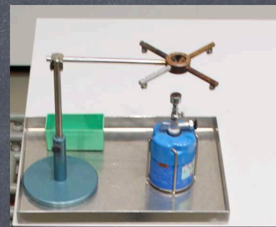
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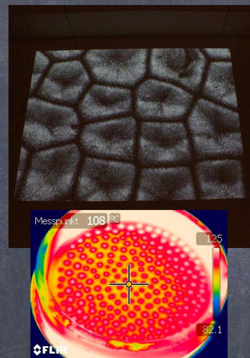
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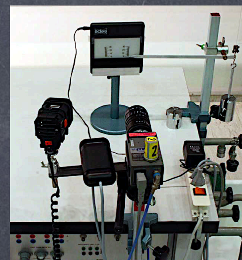
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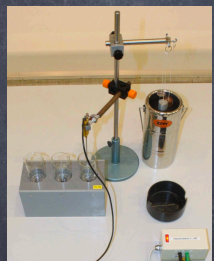
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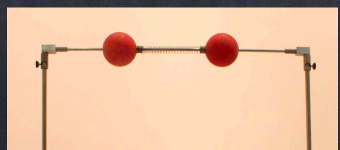
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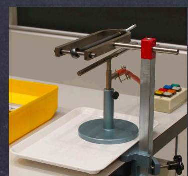
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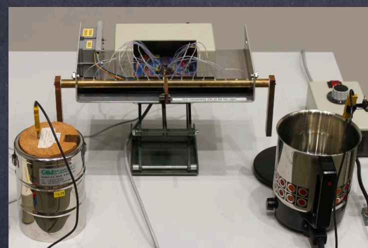
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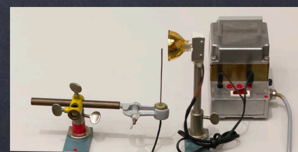
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