Measuring the Leading Order Hadronic Contribution to the *muon g-2* in the space-like region

A proposal for measuring HLO contributions from

$\mu + e \longrightarrow \mu + e$ elastic scattering

Clara Matteuzzi

Outline

- ★ **Physics motivations**
- ★ **Tools to perform the measurement**
- ★ **First testbeam at CERN**
- ★ **Plans and tentative timeline**

★ **Physics motivation**

Physics motivations

The magnetic moment of a particle with charge e and spin $\vec{s}$ is

$$\vec{\mu} = g \frac{e\hbar}{2mc} \vec{s}$$

Dirac equation predicts *g*= 2 , but radiative corrections:

$$\vec{\mu} = 2(1 + a) \frac{e\hbar}{2mc} \vec{s}$$

where

$$a = \frac{g - 2}{2}$$

"anomalous" magnetic moment

$$a_\mu = a_\mu^{QED} + a_\mu^{Weak} + a_\mu^{Had}$$

Physics motivations

$$a_\mu = \frac{g-2}{2}$$

The *muon g-2* is measured with high precision:

$$a_\mu^{exp} = 116592089(63) \times 10^{-11}$$

G.W.Bennet et al. (Muon g-2 Phys.Rev.D73 (2006)072003

It shows a long standing $\approx 4\sigma$ deviations from the Standard Model prediction:

$$a_\mu^{SM} = 116591783(35) \times 10^{-11}$$

(F.Jegerlehner, MITP Workshop, 19-23 February 2018, Mainz)

$$\Delta a_\mu (\text{exp} - \text{SM}) = 306 \pm 72 \times 10^{-11}$$

The accuracy of the SM prediction $5 \cdot 10^{-10}$

is limited by **strong interactions** effects

The present error on the leading order hadronic contribution to muon $g-2$

$$\delta a_\mu^{HLO} \simeq 4 \cdot 10^{-10}$$

constitutes the main uncertainty of the SM predictions

- *New Physics?*
- *Systematics of the measurement?*
- *Systematics of the theoretical prediction?*

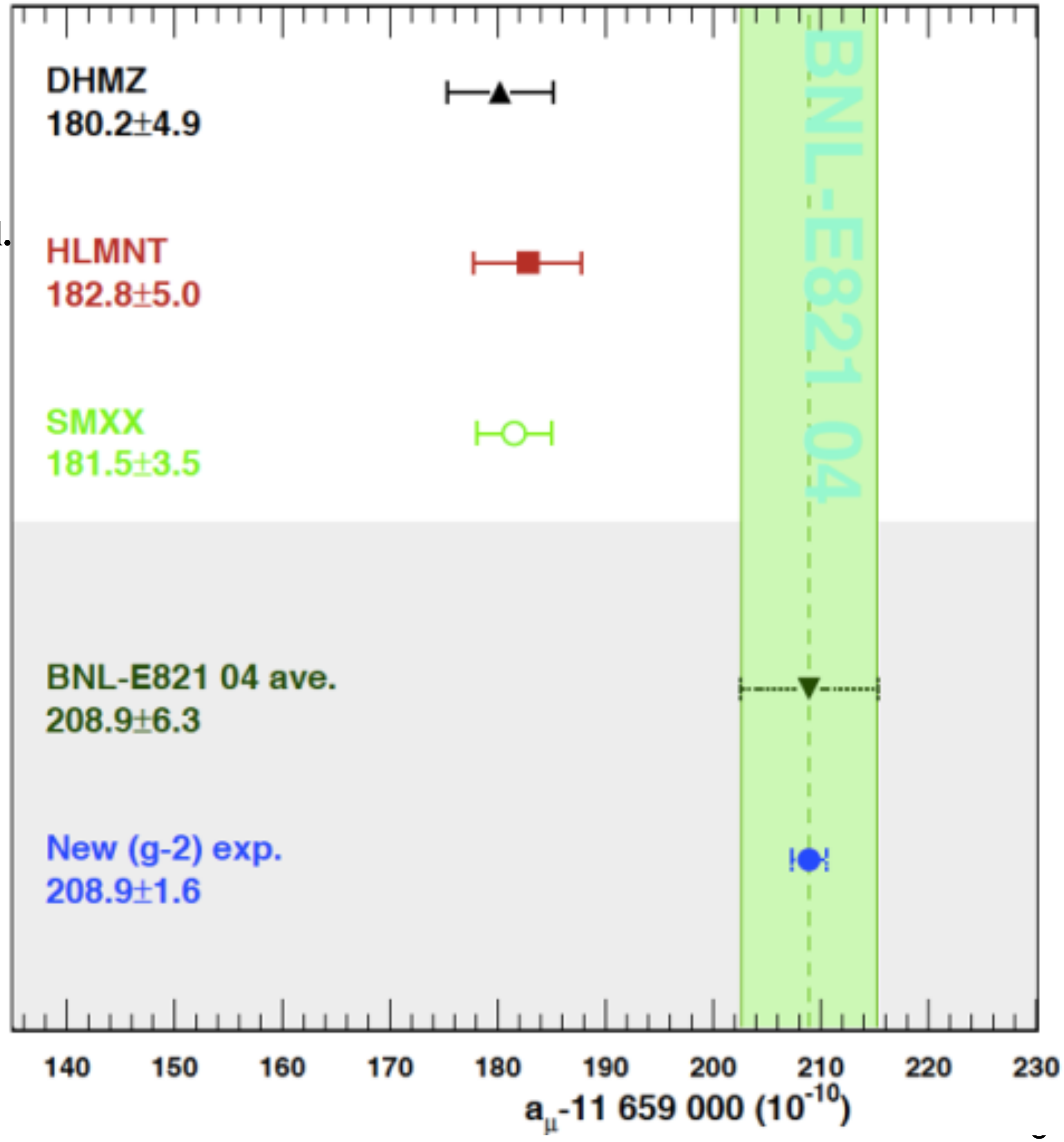
Physics motivations

T. Blum *et. al.*, "The Muon *g*-2 Theory Value: Present and Future"
arXiv:1311.2198 [hep-ph]

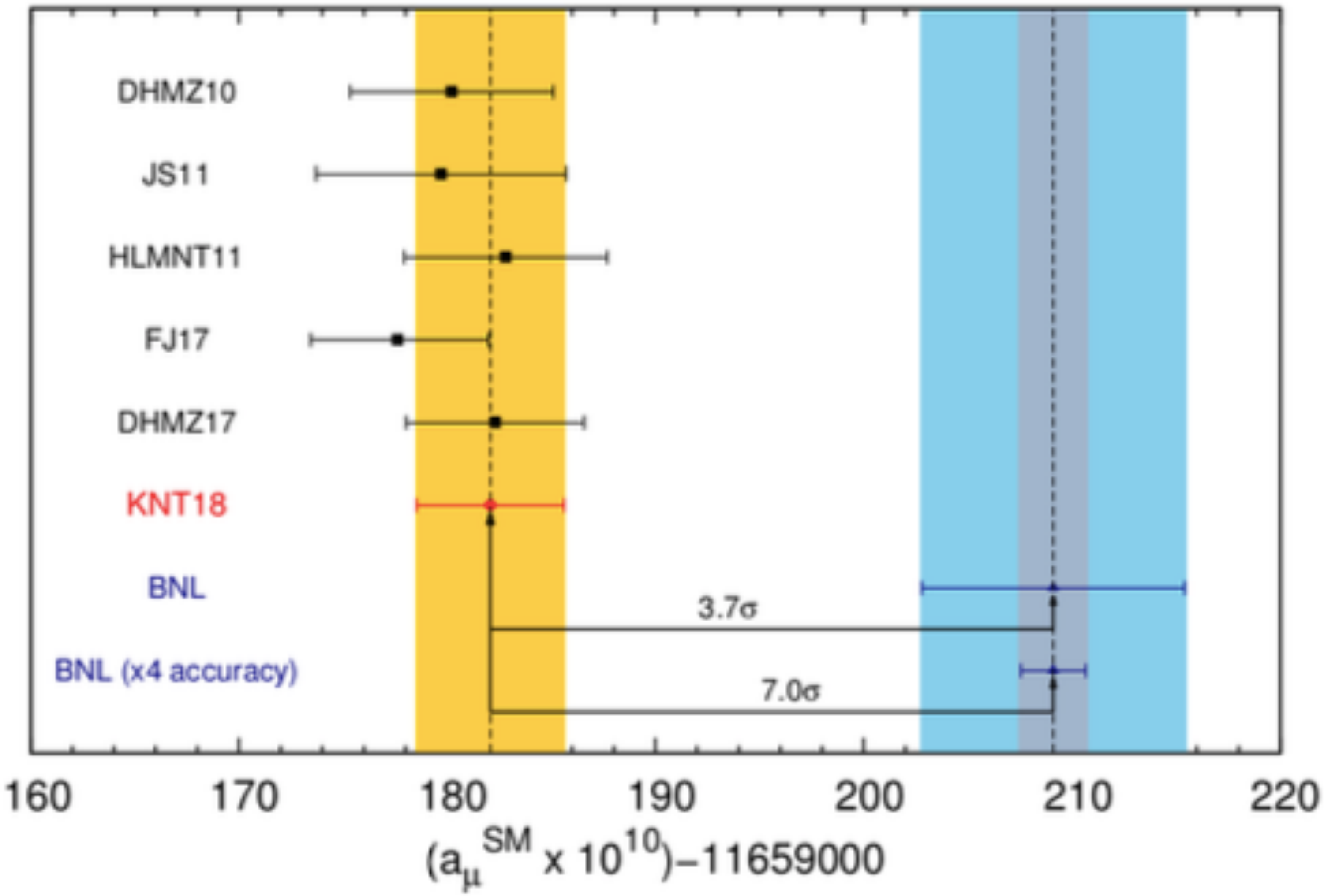
DHMZ ➔ parametrization M.Davier et al.
HLMNT ➔ K. Hagiwara et al.
SMXX ➔ average of these two values

Current experimental value of a_μ ➔

Assuming same central value and improved precision from Fermilab and J-PARC ➔



Physics motivations



Physics motivations

Recent updates :

T. Blum *et al.*, Phys.Rev.Lett. 121 022003 (2018)
A. Keshavarzi *et al.*, Phys.Rev.D 97 114025 (2018)

Theory (in units of 10^{-10}) from M. Knecht talk at Capri Workshop on FCCP2015

QED	$+11658471.9$	[T. Aoyama et al. (2015)]
HVP-LO	$+692.3(4.2)$	[M. Davier et al. (2011)]
	$+694.9(4.3)$	[K. Hagiwara et al. (2011)]
HVP-NLO	$-9.84(7)$	[K. Hagiwara et al. (2011)]
HVP-NNLO	$+1.24(1)$	[A. Kurz et al. (2014)]
HLxL	$+10.5(2.6)$	[J. Prades et al. (2009)]
	$+11.5(4.0)$	[F. Jegerlehner, A. Nyffeler (2009)]
EW 1 loop	$+19.48(1)$	[(1972)]
EW 2 loops	$-4.12(10)$	[C. Gnendiger et al. (2013)]

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (27.4 \pm 8.0) \cdot 10^{-10} \; [3.4\sigma] \quad \text{for } a_\mu^{\text{HLxL}} = (10.5 \pm 2.6) \cdot 10^{-10}, \; a_\mu^{\text{HVP-LO}} = 692.3 \pm 4.2 \cdot 10^{-10}$$

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (23.7 \pm 8.6) \cdot 10^{-10} \; [2.8\sigma] \quad \text{for } a_\mu^{\text{HLxL}} = (11.6 \pm 4.0) \cdot 10^{-10}, \; a_\mu^{\text{HVP-LO}} = 694.9 \pm 4.3 \cdot 10^{-10}$$

Is it possible, in view of the forecoming experiments at FNAL(E989) and J-PARC(E34), to reduce the dominant theoretical uncertainties (HVP-LO and HLxL) ??

Physics motivations

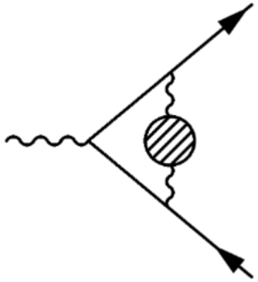
Both Fermilab and J-PARC *g-2* experiments will lower the experimental error from 0.5 ppm to ≈ 0.14 ppm in few years

Need therefore to lower the theoretical uncertainty in order to have a more precise SM prediction ➡ *more theoretical work is necessary (rad corr, lattice,..)*

The largest contribution to the theoretical uncertainty comes from the term $\Delta\alpha_{had}(t)$ **which can be measured experimentally**

★ **The standard dispersive approach**
(time-like approach)

Approach: time-like evaluation

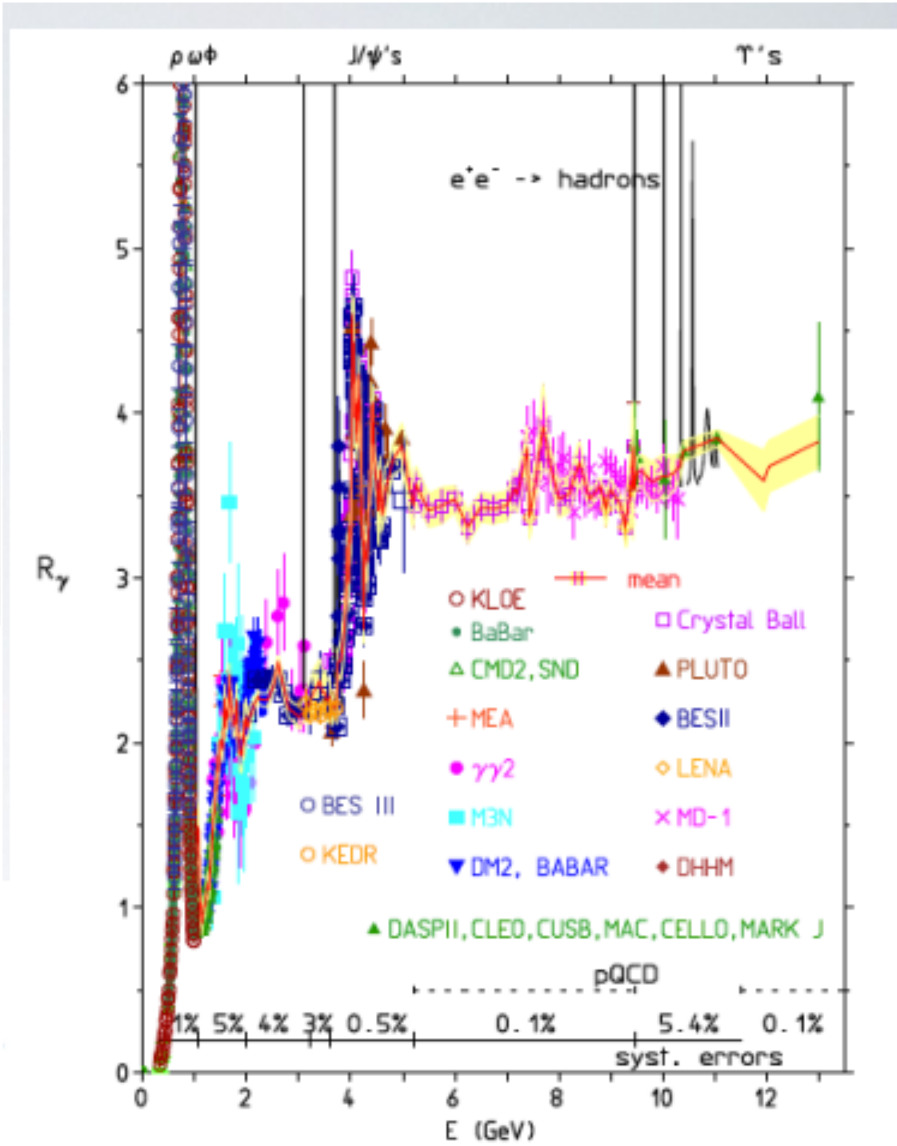


$$a_\mu^{HLO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{\hat{K}(s) R_{had}(s)}{s^2}$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\mu^2}}$$

$$R_{had}(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Combination of many exclusive channels



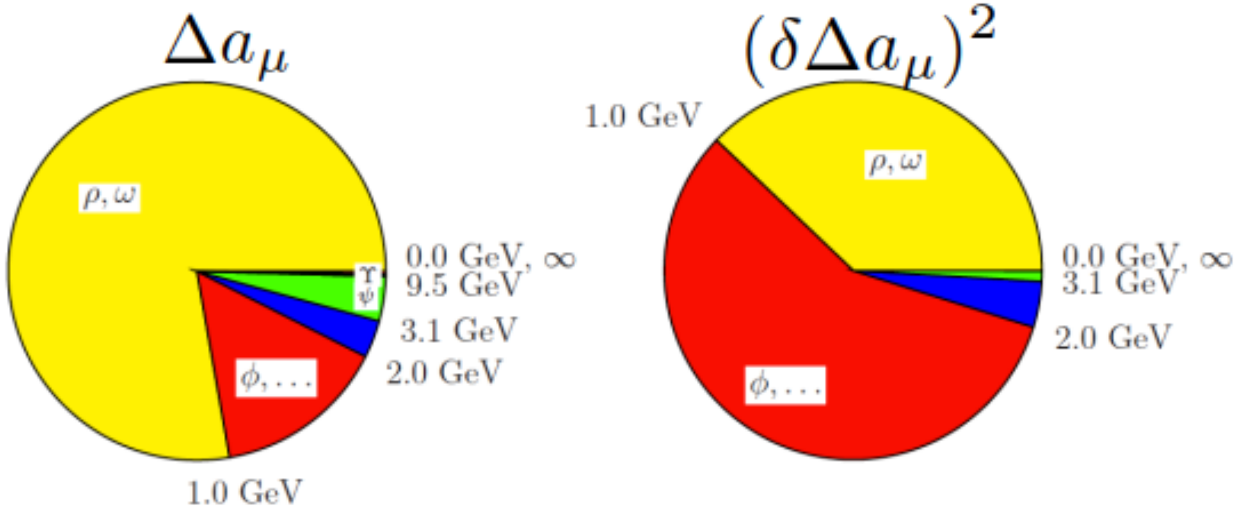
$a_\mu^{HLO} = 6870\ (42)_{tot} \times 10^{-11}$ F. Jegerlehner, arXiv:1511.04473 (includes BESIII 2π)

$= 6923\ (42)_{tot} \times 10^{-11}$ Davier et al, EPJ C71 (2011) 1515

$= 6949\ (37)_{exp}\ (21)_{rad} \times 10^{-11}$ Hagiwara et al, JPG 38 (2011) 085003

Approach: time-like evaluation

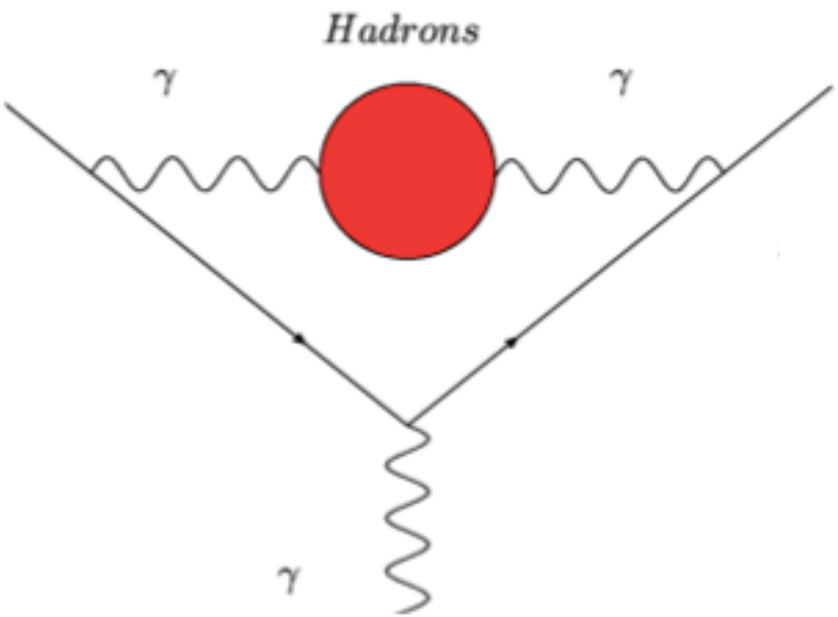
Contributions to Δa_μ



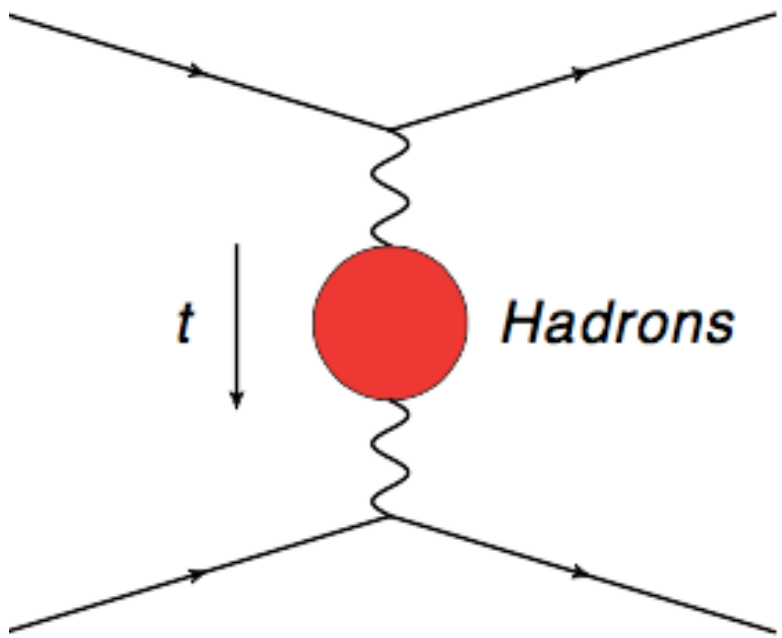
from F. Jegerlehner talk in Frascati March 23, 2016

★ **An alternative approach**
(space-like approach)

From :



To :



Approach: space-like evaluation

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx(x-1)\bar{\Pi}_{\mathrm{had}}(t(x)) = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{\mathrm{had}}(t(x))$$

$$t(x) = -\frac{x^2 m_\mu^2}{1-x} \qquad t = \begin{cases} 0^- & \text{for } x \to 0^+ \\ -\infty & \text{for } x \to 1^- \end{cases}$$

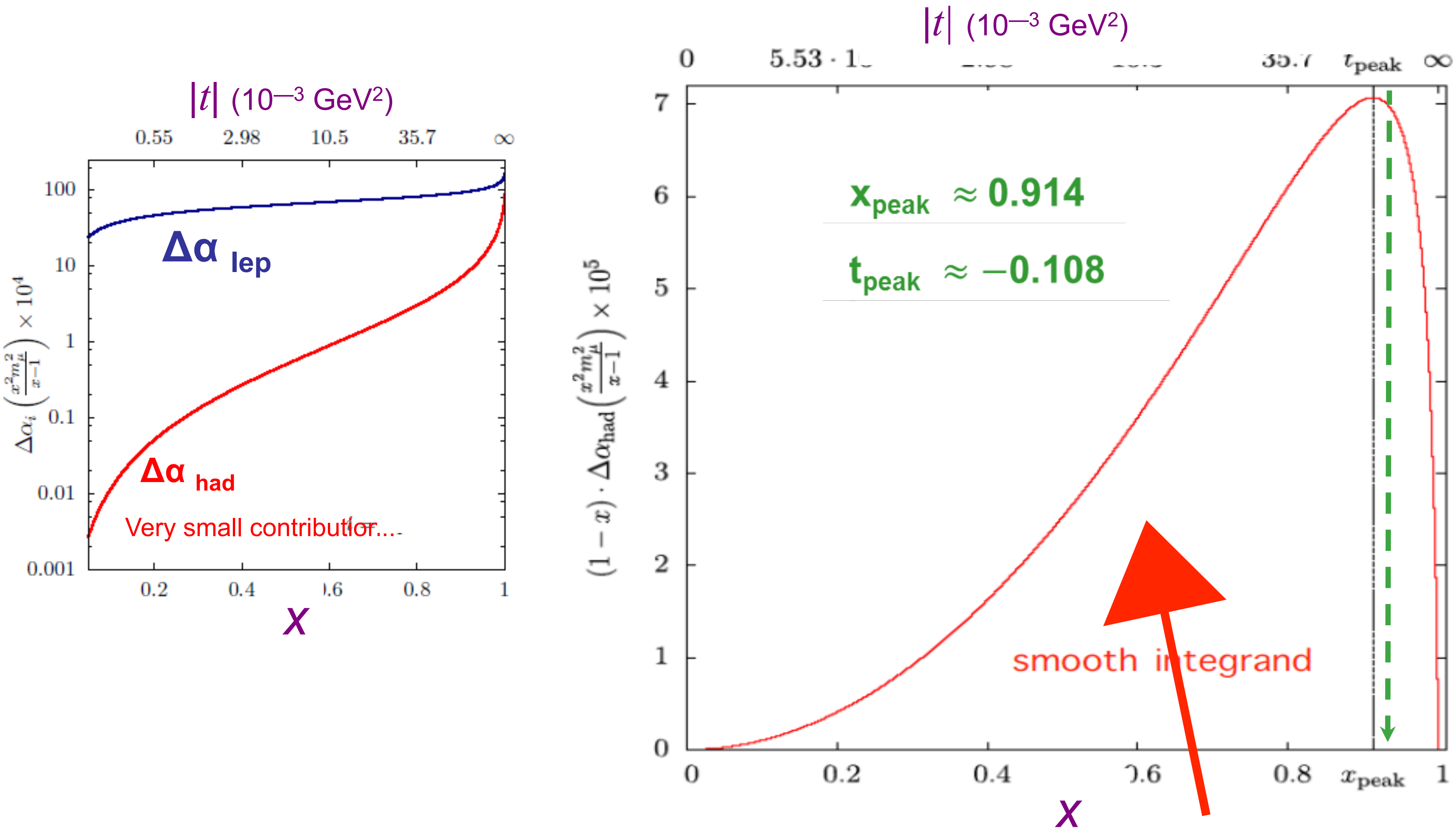
$$\alpha(t) = \frac{\alpha}{1 - \Delta\alpha_{\mathrm{other}}(t) - \Delta\alpha_{\mathrm{had}}(t)}$$

$= \Delta\alpha_{\mathrm{leptonic}} + \Delta\alpha_{gb} + \Delta\alpha_{\mathrm{top}}$

★ **Strategy:**

- measure $\Delta\alpha_{\mathrm{had}}(t)$ in the reachable experimental kinematic range
- fit $\Delta\alpha_{\mathrm{had}}(t)$
- get large |t| values from theory
- get the integrand function and the value of a_μ^{HLO}

Approach: space-like evaluation

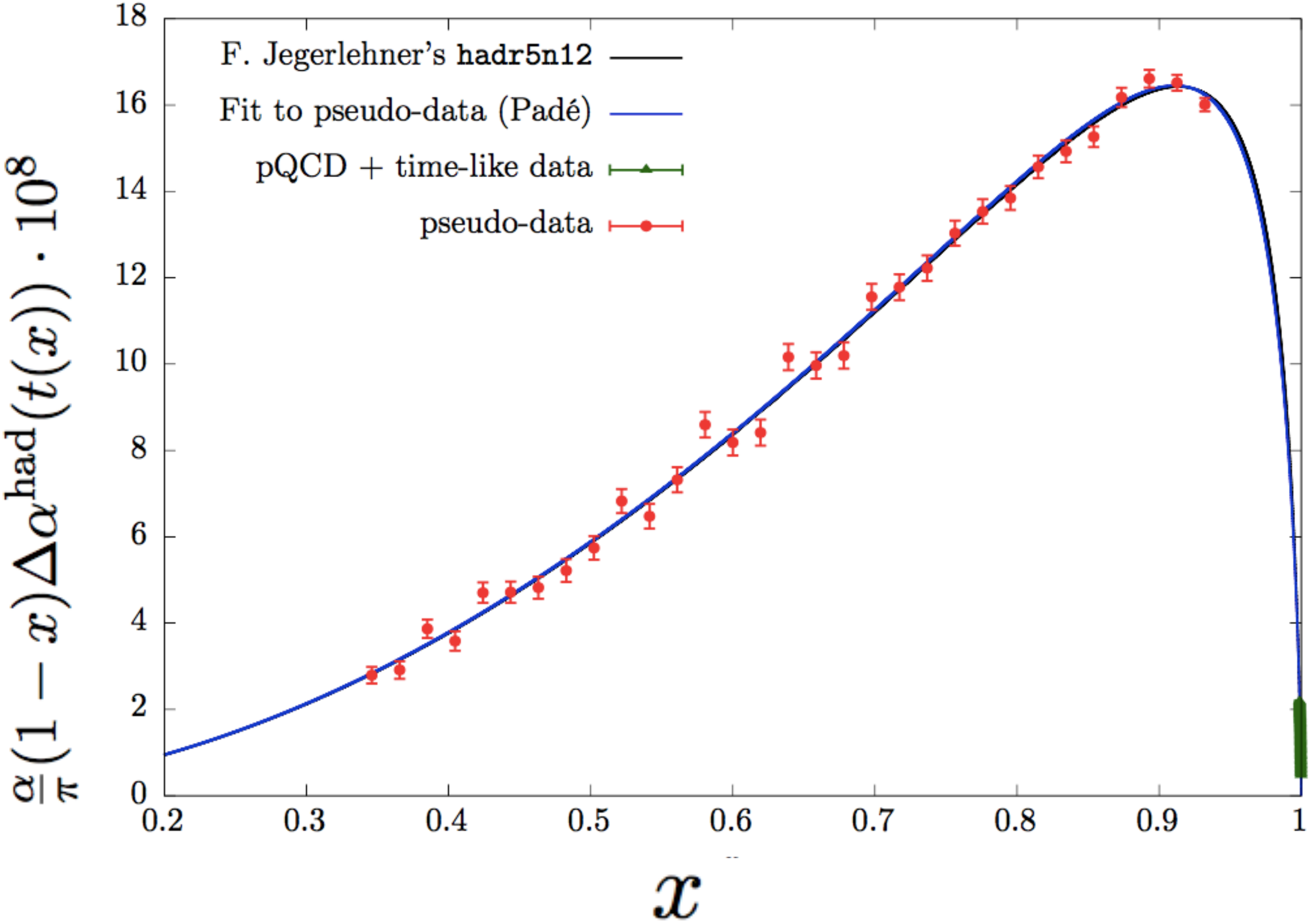


integrand function $(1-x)\Delta\alpha_{had}(t(x))$

a_{HAD} is the total area under this curve

★ Fit $\Delta\alpha_{had}(t)$ from :

(From C. Carloni-Calame)



To be competitive with the current evaluations, $\Delta\alpha_{had}(t)$ needs to be measured at the % level

Genesis of the proposal …..

1. Running of $\alpha_{e.m.}$ with Bhabha scattering

The running of the electromagnetic coupling α in small-angle Bhabha scattering

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2. Extract HLO from the running of $\alpha_{e.m}$

A new approach to evaluate the leading hadronic corrections to the muon g-2 ☆

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Measuring the leading hadronic contribution to the muon g-2 via μe scattering

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Received: date / Accepted: date

3. measure HLO from the $\mu + e \rightarrow \mu + e$ elastic scattering

★ **Why $\mu + e^- \rightarrow \mu + e^-$**

Why $\mu + e \to \mu + e$?

Muon scattering on atomic electrons looks an ideal process:

- It is a pure *t*-channel process:

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt}\left|\frac{\alpha(t)}{\alpha}\right|^2 \qquad \Delta\alpha_{had}(t_{peak}) = \mathcal{O}(10^{-3})$$

- A high intensity ($\sim 1.3 \times 10^7\ \mu/s$) muon beam is available in the North Area at CERN with **E_{beam} ~ 150 GeV**

- The high boosted kinematics allows to access the full kinematic range with small transverse dimension

$$-0.143 \lesssim t < 0 \text{ GeV}^2 \qquad 0 < x \lesssim 0.93$$ ➡ *(it spans the peak)*

- The same detector can cover **signal (x > 0.4 – 0.5)** and **normalization (x $\leq$ 0.3)** regions

The kinematics

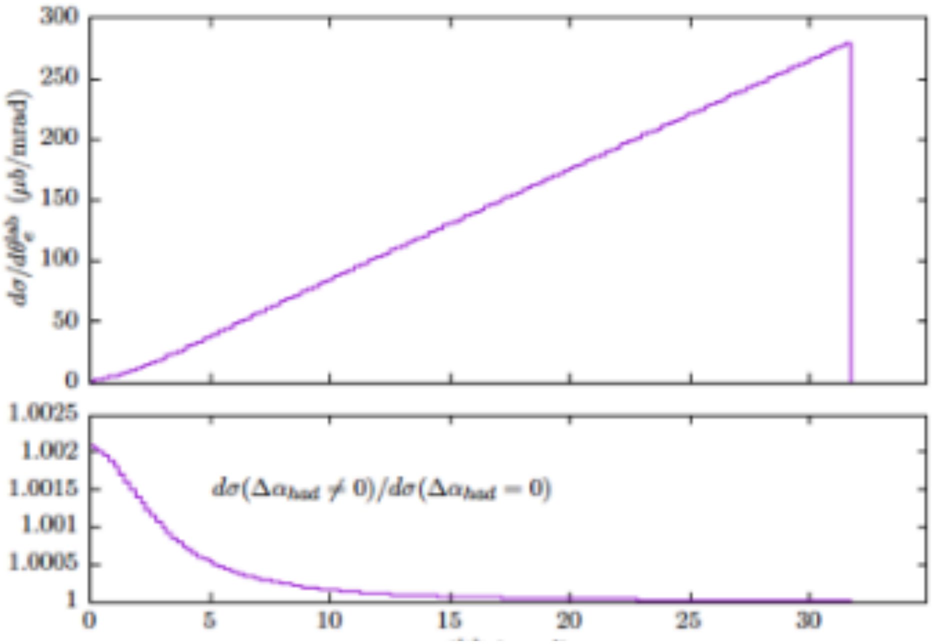
(from C.Calame)

- the $2 \to 2$ kinematics reads

$$t = 2m_e^2 - 2m_e E_e, \qquad s = m_\mu^2 + m_e^2 + 2m_e E_\mu^i \qquad r \equiv \frac{\sqrt{(E_\mu^i)^2 - m_\mu^2}}{E_\mu^i + m_e}$$

$$E_e = m_e \frac{1 + r^2 c_e^2}{1 - r^2 c_e^2}, \qquad \theta_e = \arccos\left(\frac{1}{r}\sqrt{\frac{E_e - m_e}{E_e + m_e}}\right) \qquad c_e \equiv \cos\theta_e$$

- $0 < \theta_e < 31.85$ mrad $\longleftrightarrow$ $139.8 > E_e > 1$ GeV $\longleftrightarrow$ $-0.143 < t < -10^{-3}$ GeV^2



θ$_e$*(mrad)*

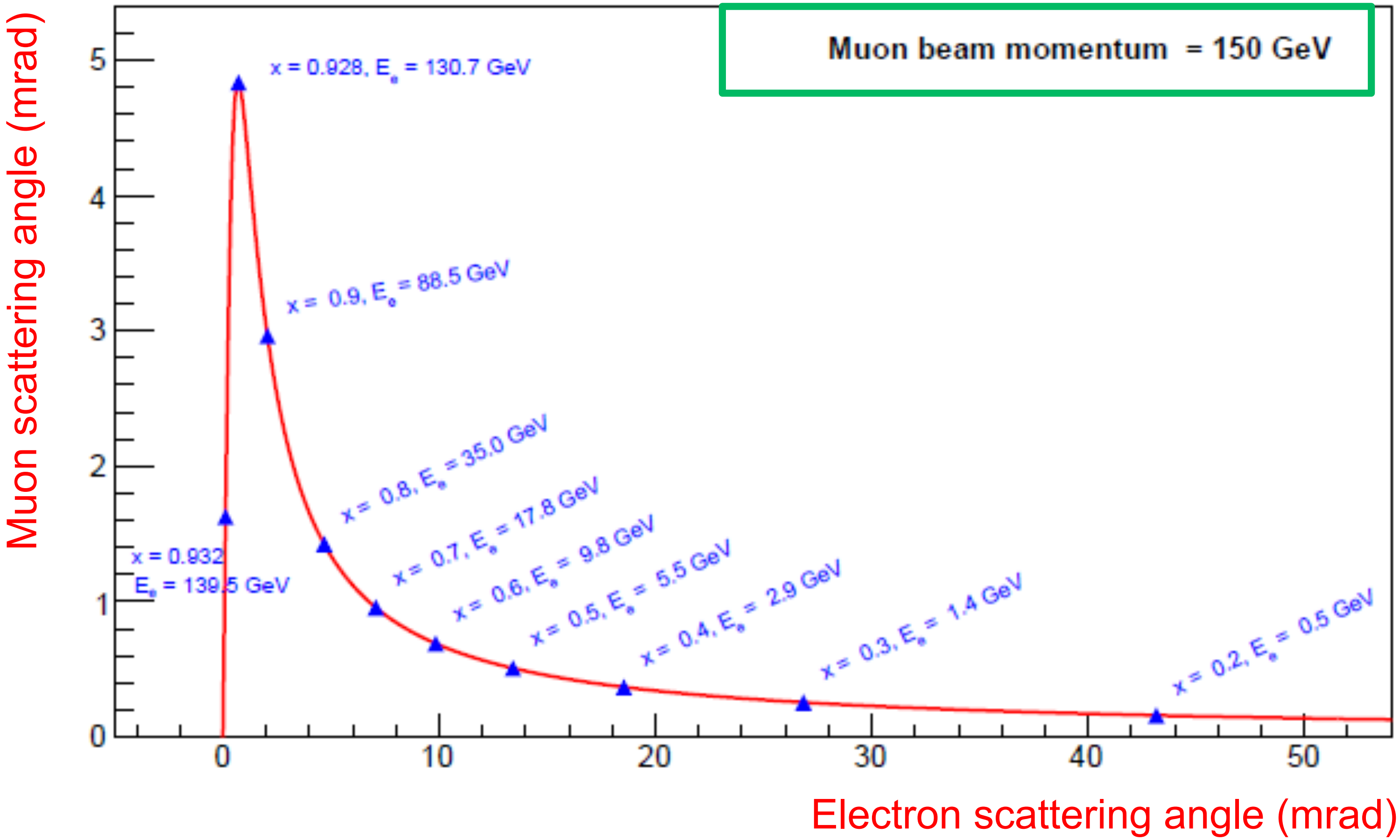
→ differential cross-section at LO (including vacuum polarization) as a function of θ_e

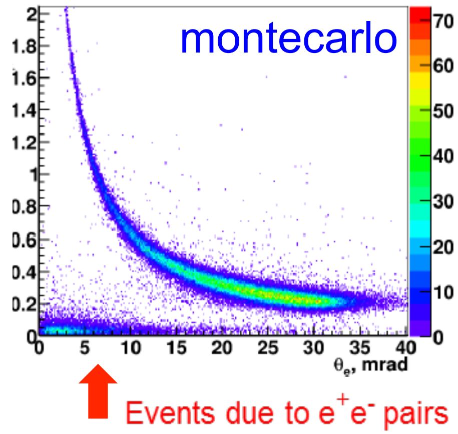
→ effect due to $\Delta\alpha_{had}(t)$

→ for instance the region $\theta_e > 20 \div 25$ mrad can be used as normalization

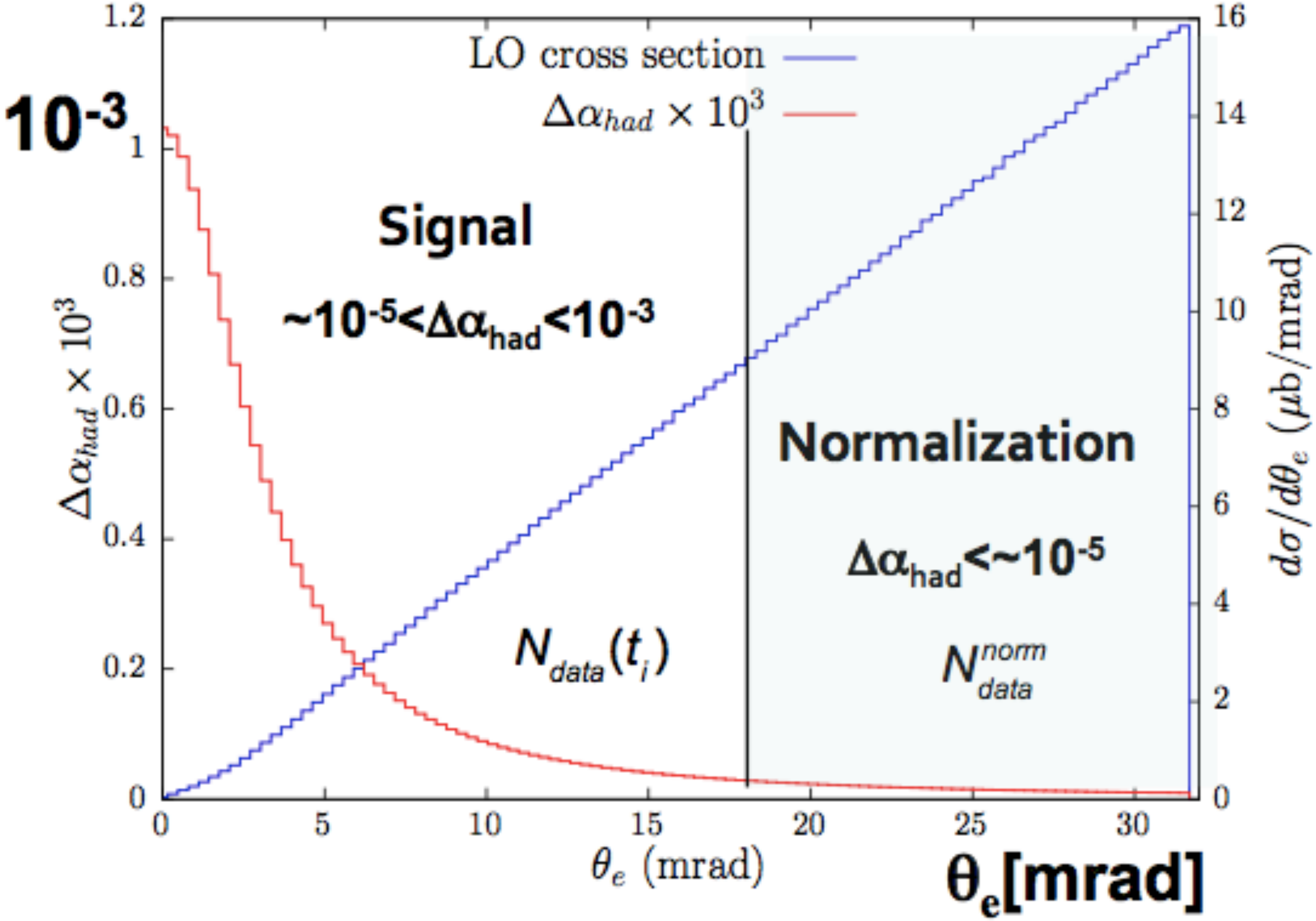
The kinematics: *correlation curve*

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$





- The constraint is useful to select elastic events, reject background and reduce systematics in *t* determination
- Below 2-3 mrad *μ* and *e* overlap, to be resolved by *μ/e* identification
- Multiple scattering breaks the correlation: simulation and data will help to optimize the detector and reduce the systematics



LO cross section
$\Delta\alpha_{had} \times 10^3$
10^{-3}
Signal
$\sim 10^{-5} < \Delta\alpha_{had} < 10^{-3}$
Normalization
$\Delta\alpha_{had} < \sim 10^{-5}$
$N_{data}(t_i)$
N_{data}^{norm}
$\Delta\alpha_{had} \times 10^3$
$d\sigma/d\theta_e$ (μb/mrad)
θ_e (mrad)
θ_e[mrad]

Tools to do the measurement

A high energy muon beam *(must cover the t range needed)*

the beam **M2 at CERN** has the characteristics **($E_\mu \approx 150$ GeV, 1.3×10^7 μ/s)** adequate for such a measurement.

The target : atomic electrons must be provided by a light material,

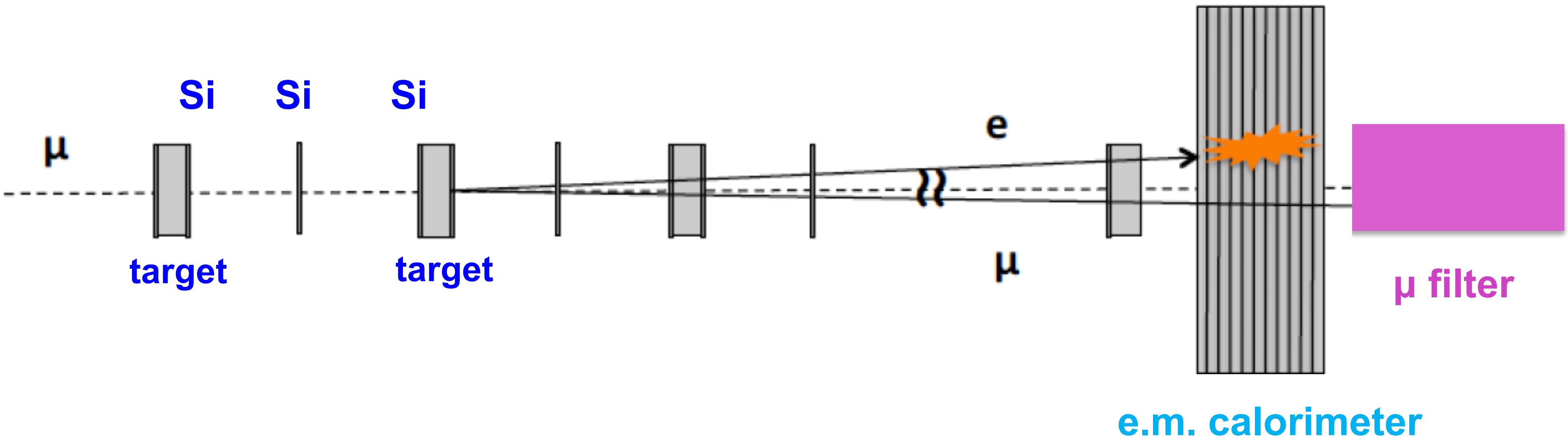
to minimize the e.m. interactions inside the target, but at the same time must provide a high enough number of target electrons
Berillium (or eventually Carbon)

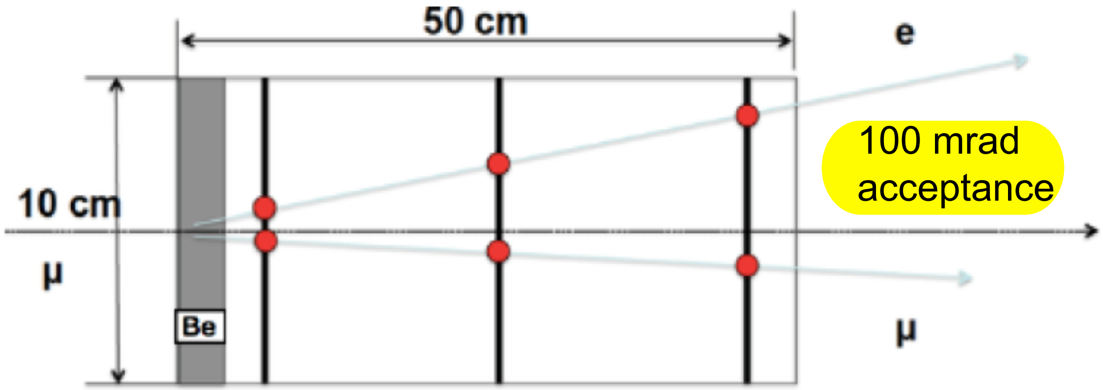
The detector setup:

- ➡ a modular target made by 60 layers of Be (C) 1 (0.8) cm thick, sandwiched in layers of Si tracking planes.
- ➡ Need to measure very precisely the angles of the outcoming muons and electrons (to exploit kinematical correlation of the μ-e collision)
- ➡ Need to measure energy and direction of the incoming muon (a la COMPASS or NA62 GTRK)
- ➡ a simple PID (e.m. calorimeter and muon system) will be necessary

Tools to do the measurement

Target elements are sandwiched between Si planes and spaced by ~ 50 cm air





Not on scale!

Tools to do the measurement

Statistics

Expected statistics achievable assuming:

the μ beam **M2** with 1.3×10^{7} μ/s , running time 2×10^{7} s/yr

60 layers of 10 mm Be target → 60 cm Be

Lumi ~ 0.8×10^{7} nb^{-1} /yr

⟶ ~ **2×10^{12}** events /yr (will allow to have a statistical precision of 0.3 % on a_{μ}^{HLO} in two years running)

Systematics

many effects will have to be under control:

efficiencies and stability (uniformity,acceptance, tracking, trigger, PID)

alignment and positioning along the beam of the Si planes

uncertainties in vertex location, incoming muon momentum,

effect of multiple scattering *(different in "control" and "signal" regions)*

??????? many others, can be studied with data themselves

Theory

to extract Δα(t) from this measurement, the SM predictions must be known at the NNLO

Tools to do the measurement

This is an experiment where the main issue is to control the systematic error at the same level as the statistical one

➡ *Important contribution identified is the multiple scattering of low energy electrons ($E_e \approx$ few GeV)*

To demonstrate the feasibility of the experiment, we started a testbeams campaign

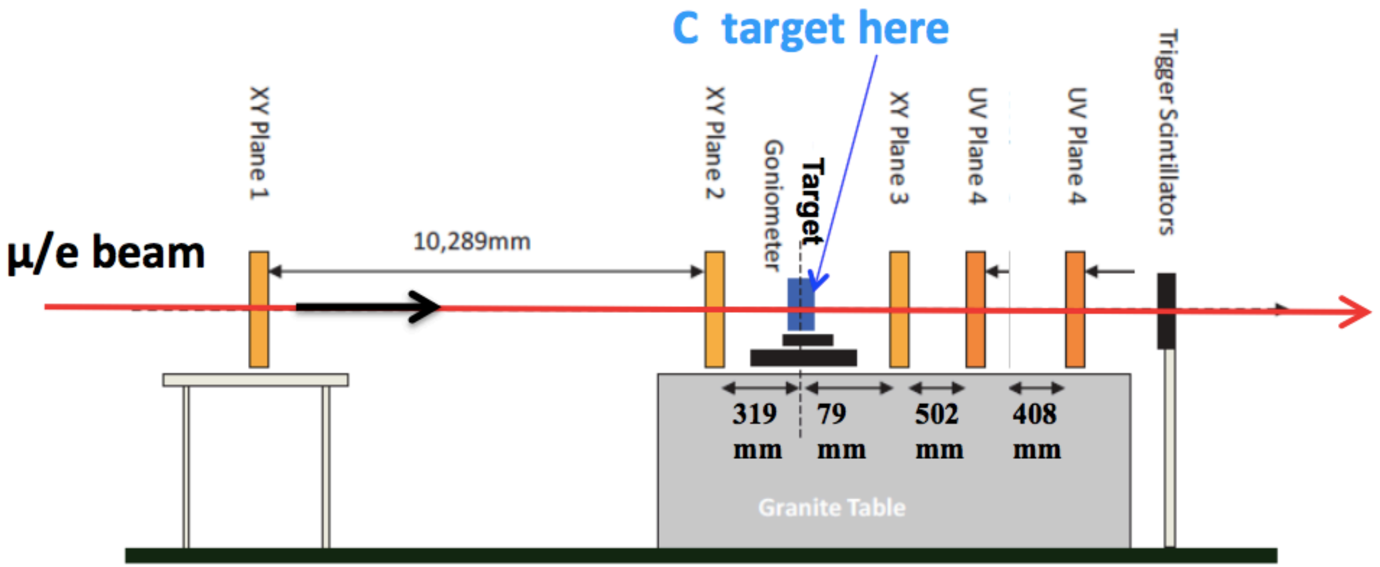
★ Testbeams results

Testbeam in 2017

Used existing setup from Imperial College :

5 Si planes, 2 before and 3 after the target, 3.8x3.8 cm2

as is it the setup achieves 5.2 μrad, limited by the MS in the Si



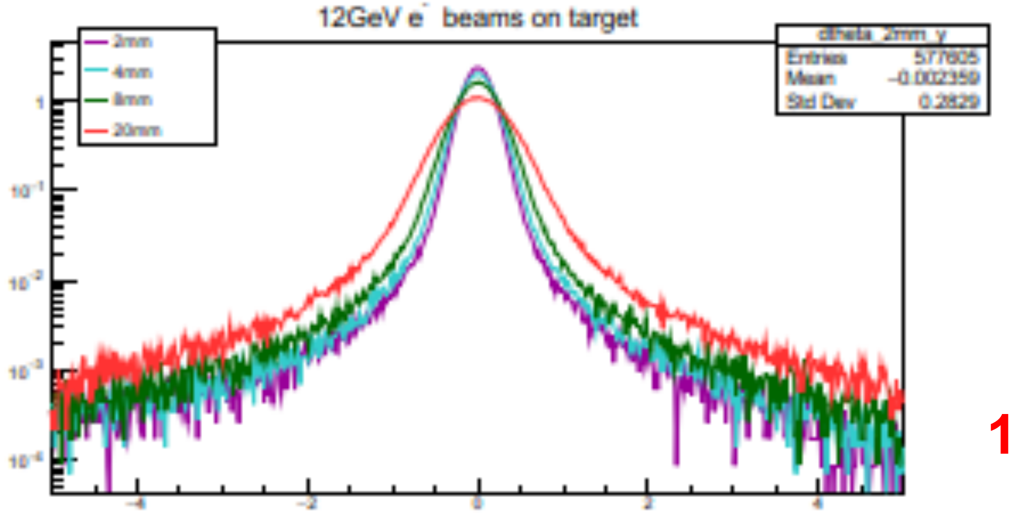
Data taken with electrons and muons and with different targets thicknesses

Aim: study MS of electrons and first look at elastic events

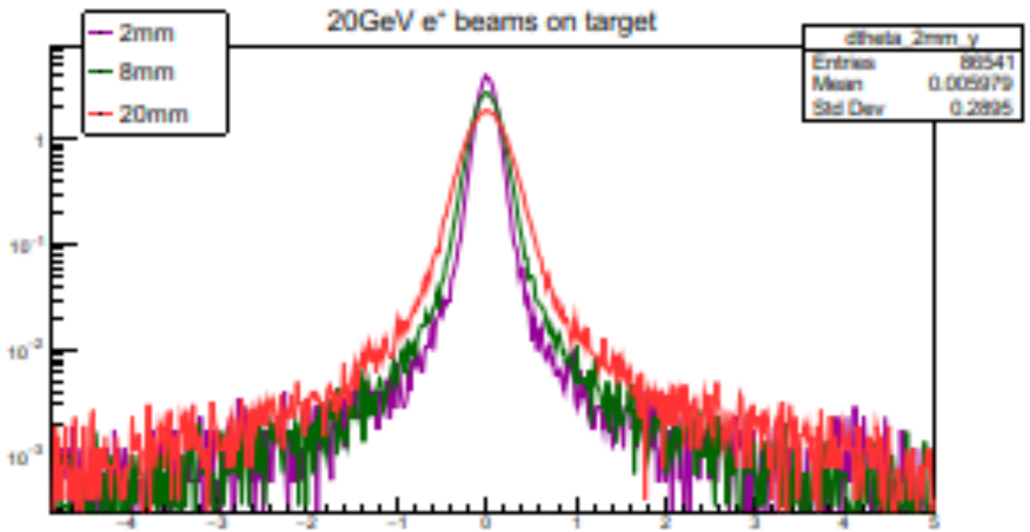
Testbeam in 2017 *(Multiple Scattering study)*

Data taken with
- Graphite targets of 4, 8, 20 mm
- Electron energies 12 and 20 GeV
- Muons of 160 GeV

(plots from M. Bonanomi thesis)



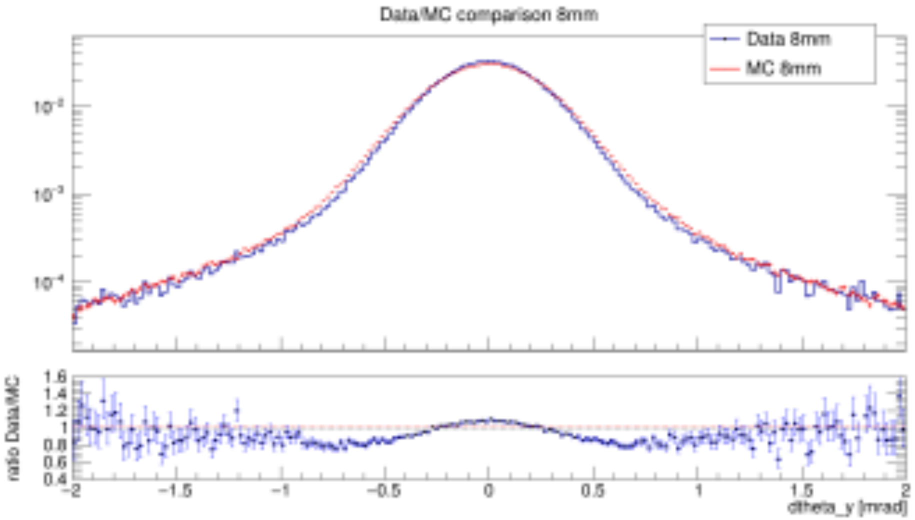
12 GeV electrons



20 GeV electrons

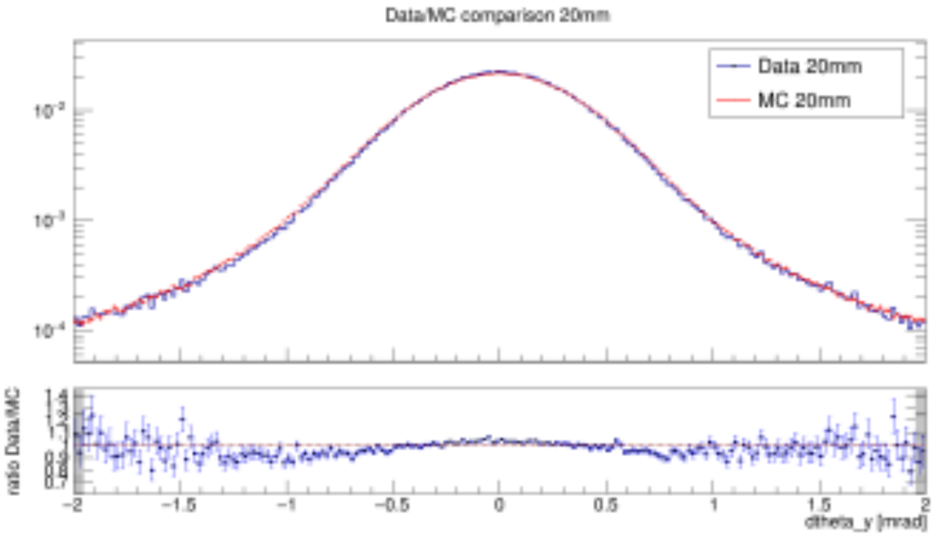
Testbeam in 2017 *(Multiple Scattering study)*

(plots from M. Bonanomi thesis)



12 GeV electrons, 8 mm graphite
Comparison GEANT/DATA

	$\theta_{MS}(20GeV,\ e^-)$ [mrad]	$\theta_{MS}(12GeV,\ e^-)$ [mrad]
2 mm		0.103 ± 0.008 (0.1050)
4 mm	0.0879 ± 0.0004 (0.0891)	0.147 ± 0.006 (0.1485)
8 mm	0.1268 ± 0.0003 (0.1260)	0.212 ± 0.005 (0.2100)
20 mm		0.3205 ± 0.005 (0.3320)



12 GeV electrons, 20 mm graphite
GEANT/DATA

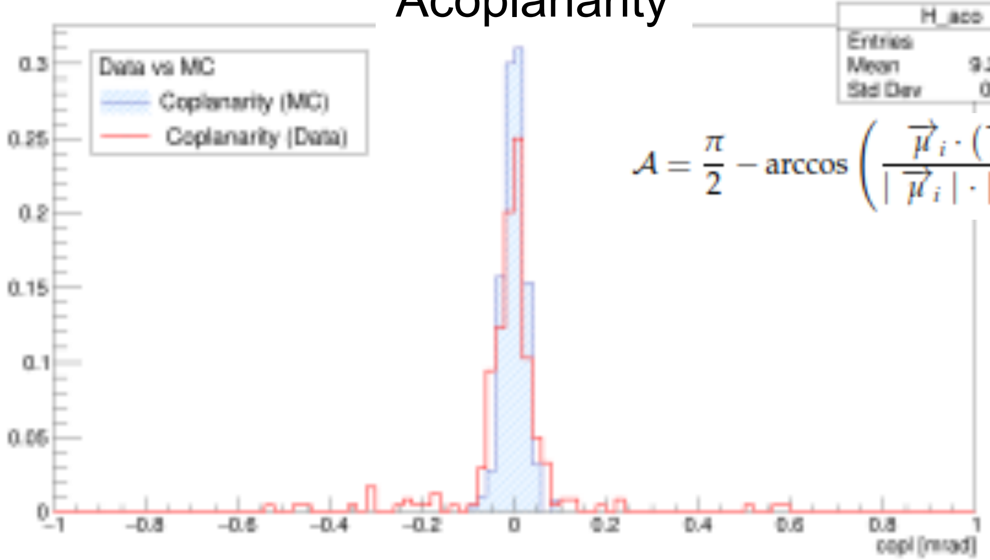
Testbeam in 2017 *(study of $\mu + e^- \rightarrow \mu + e^-$)*

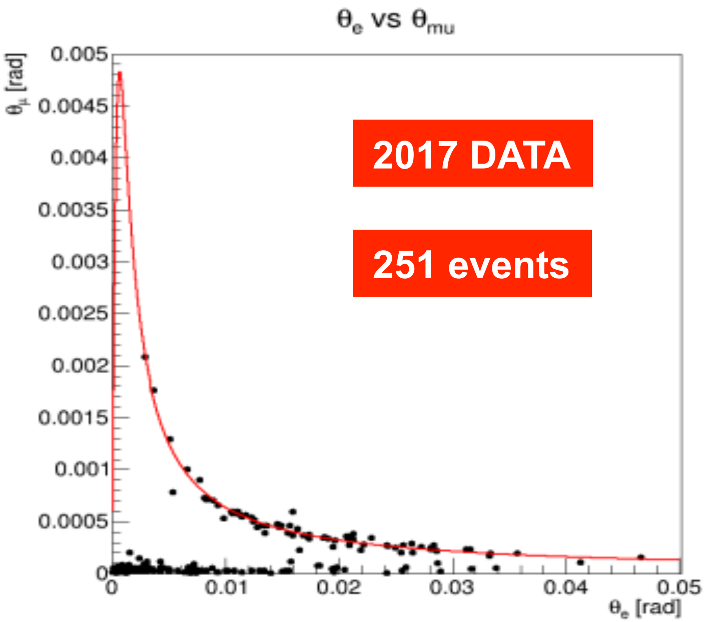
Data taken with Muons of 160 GeV

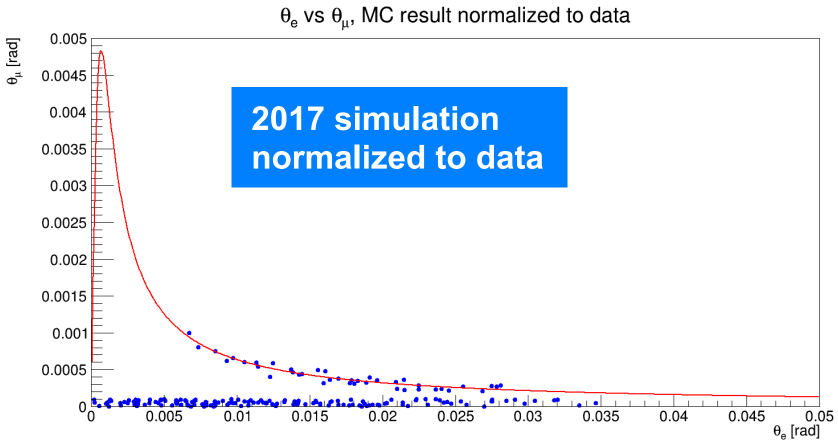
(plots from M. Bonanomi thesis)

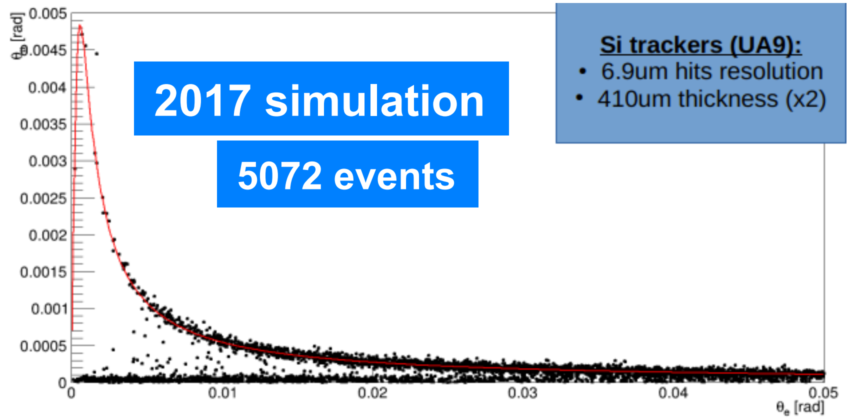
Acoplanarity

$$\mathcal{A} = \frac{\pi}{2} - \arccos\left(\frac{\vec{\mu}_i \cdot (\vec{\mu}_o \times \vec{e}_o)}{|\vec{\mu}_i| \cdot |\vec{\mu}_o \times \vec{e}_o|}\right)$$









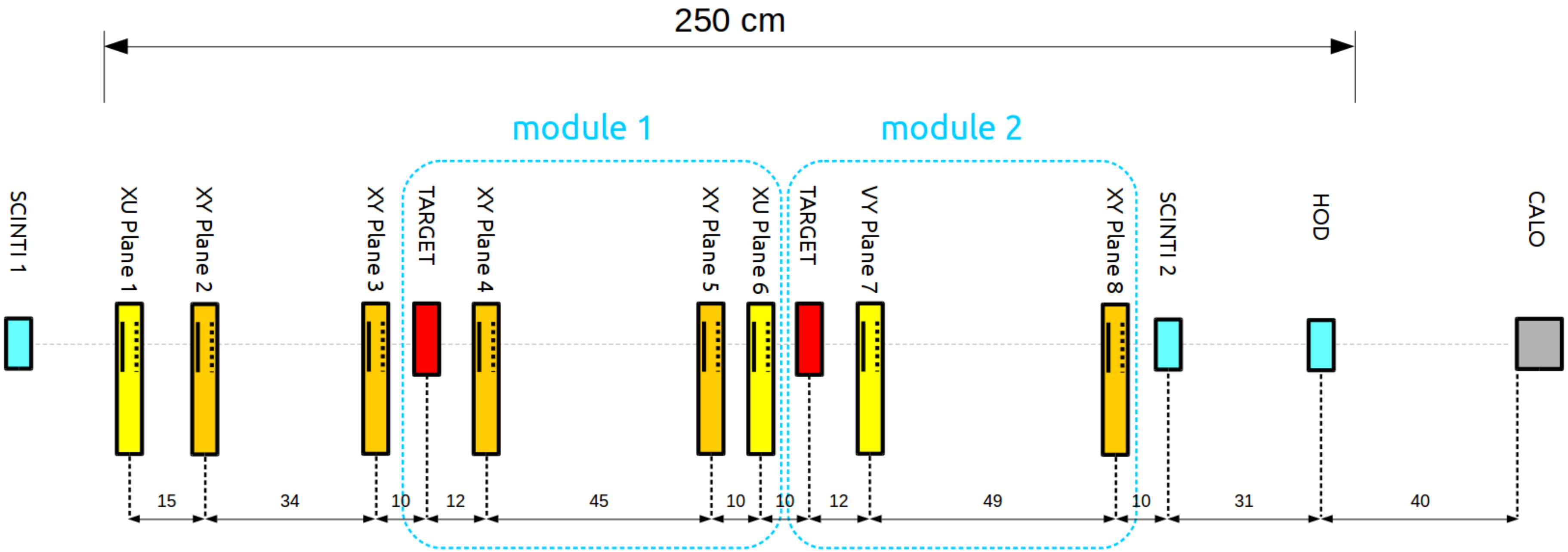
Si trackers (UA9):
- 6.9um hits resolution
- 410um thickness (x2)

5072 evts

Testbeam in 2018

Two ‘modules’ :

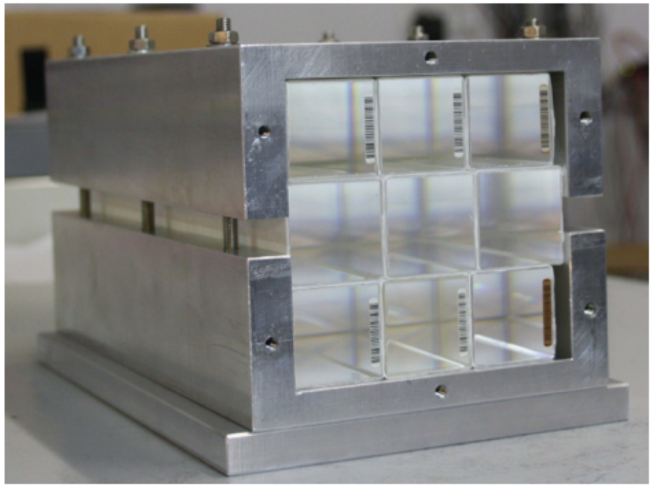
8 Si planes 9.5 x 9.5 cm , 2 C targets (8 mm thick)
e.m. calorimeter (CMS crystals)



Running with muons behind COMPASS detector

Testbeam in 2018

Setup located in the North Area behind COMPASS detector



Array of 3 x 3 crystals from CMS
PbWO , $25X_o$, 9.9 cm^2

Testbeam in 2018

- **The setup has been located downstream COMPASS**

  behind the Tungsten hadrons filter
- **Aim to measure muon – electron elastic scattering**
  - ***Using muons from pions decays (hadron beam) with an estimated beam momentum p = (187±7) GeV***
  - ***To measure the correlation between the scattering angles: muon angle vs the electron angle;***
  - ***Electron energy vs the electron angle correlation and PID.***
- **The detector consists of:**
  - **Tracking system: stations equipped with the AGILE silicon strip sensors: 400 micron thick, single sided, about 40 micron intrinsic hit resolution.**
  - **Electromagnetic calorimeter: 3x3 cell matrix.**

Testbeam in 2018

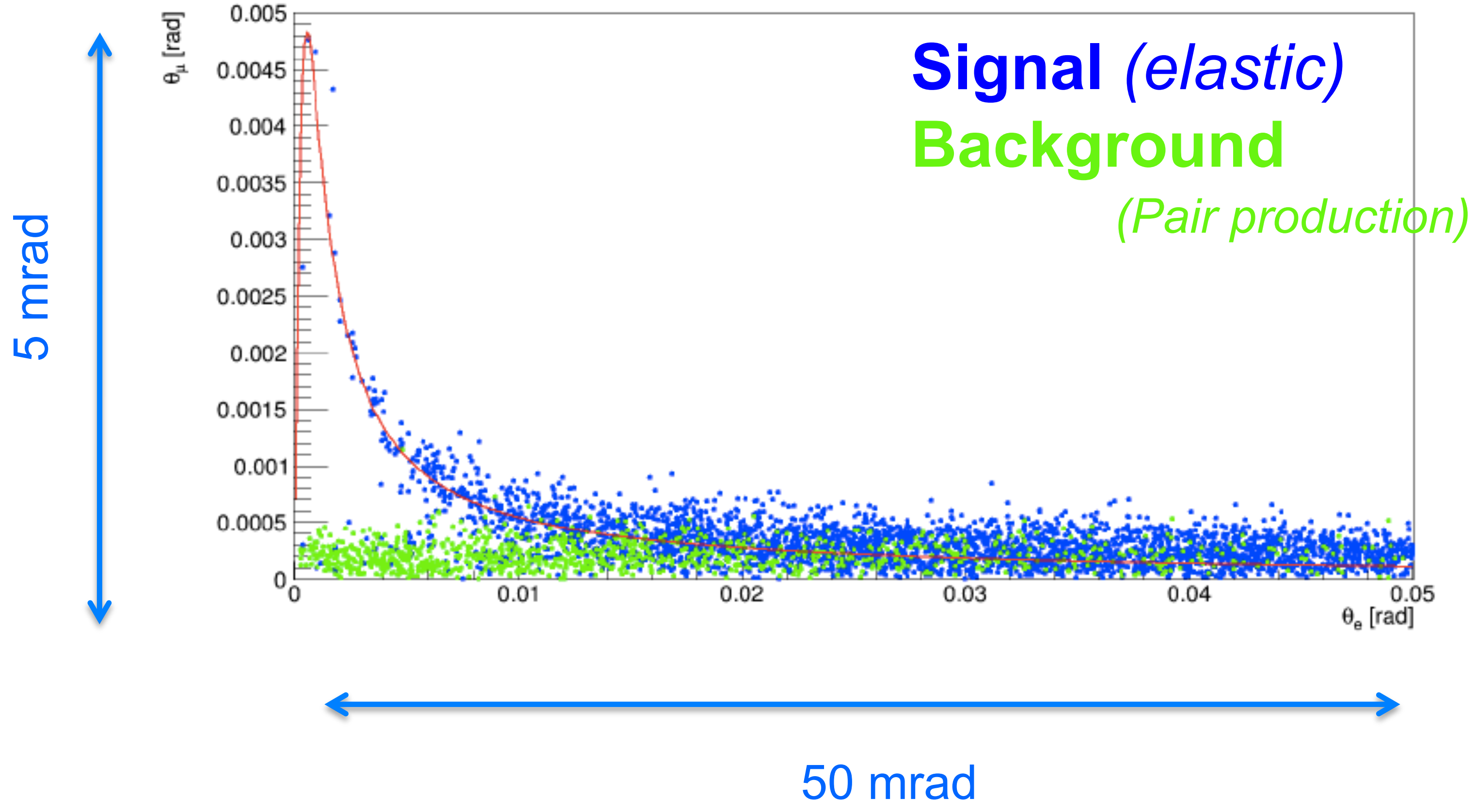
θ_μ vs θ_e

40μm hit resolution

Elasticity curve: beam momentum at 187 GeV

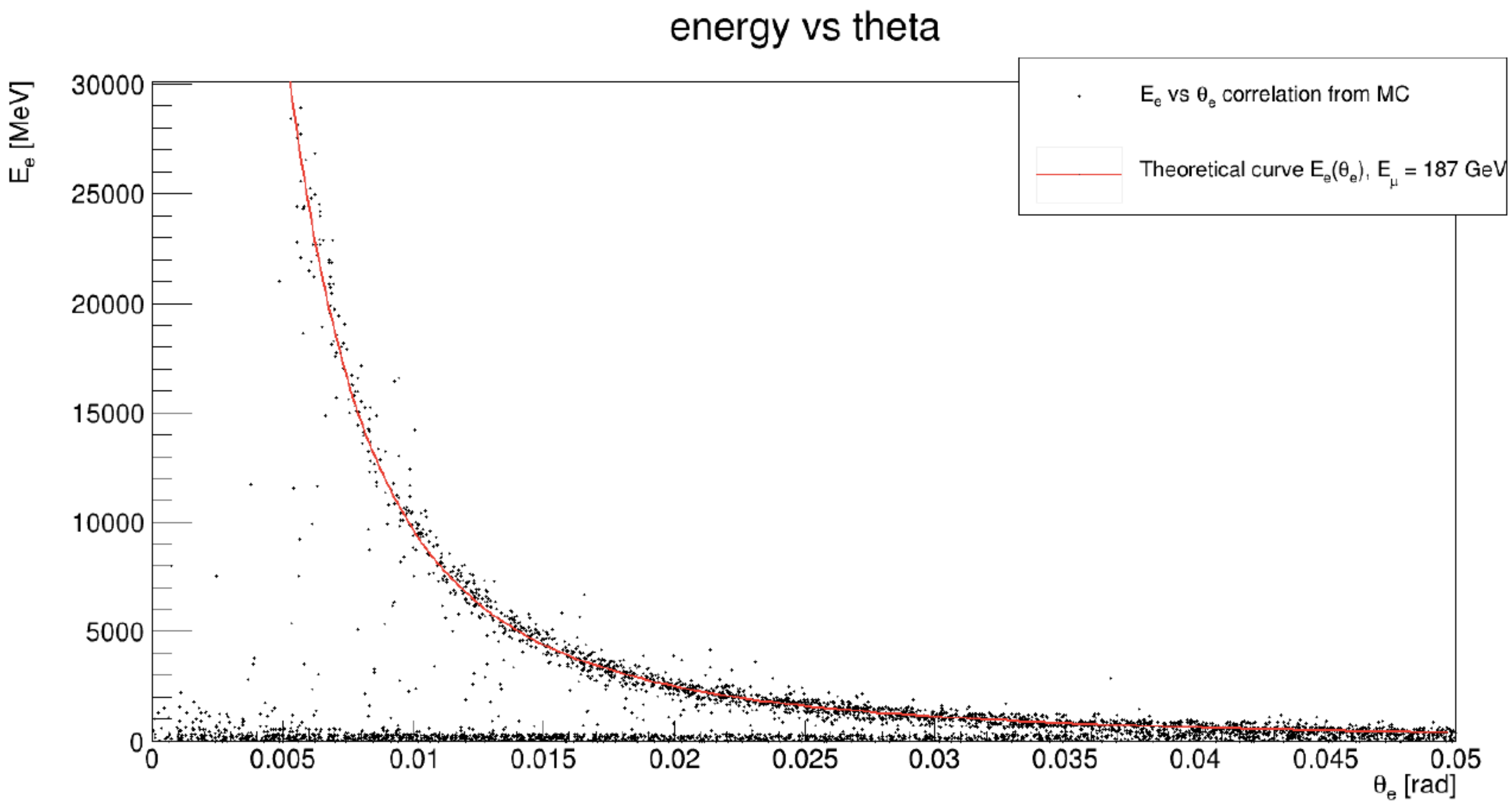
Tracking algorithm applied.
150 million incoming muons → 5742 reconstructed events

Simulation GEANT4



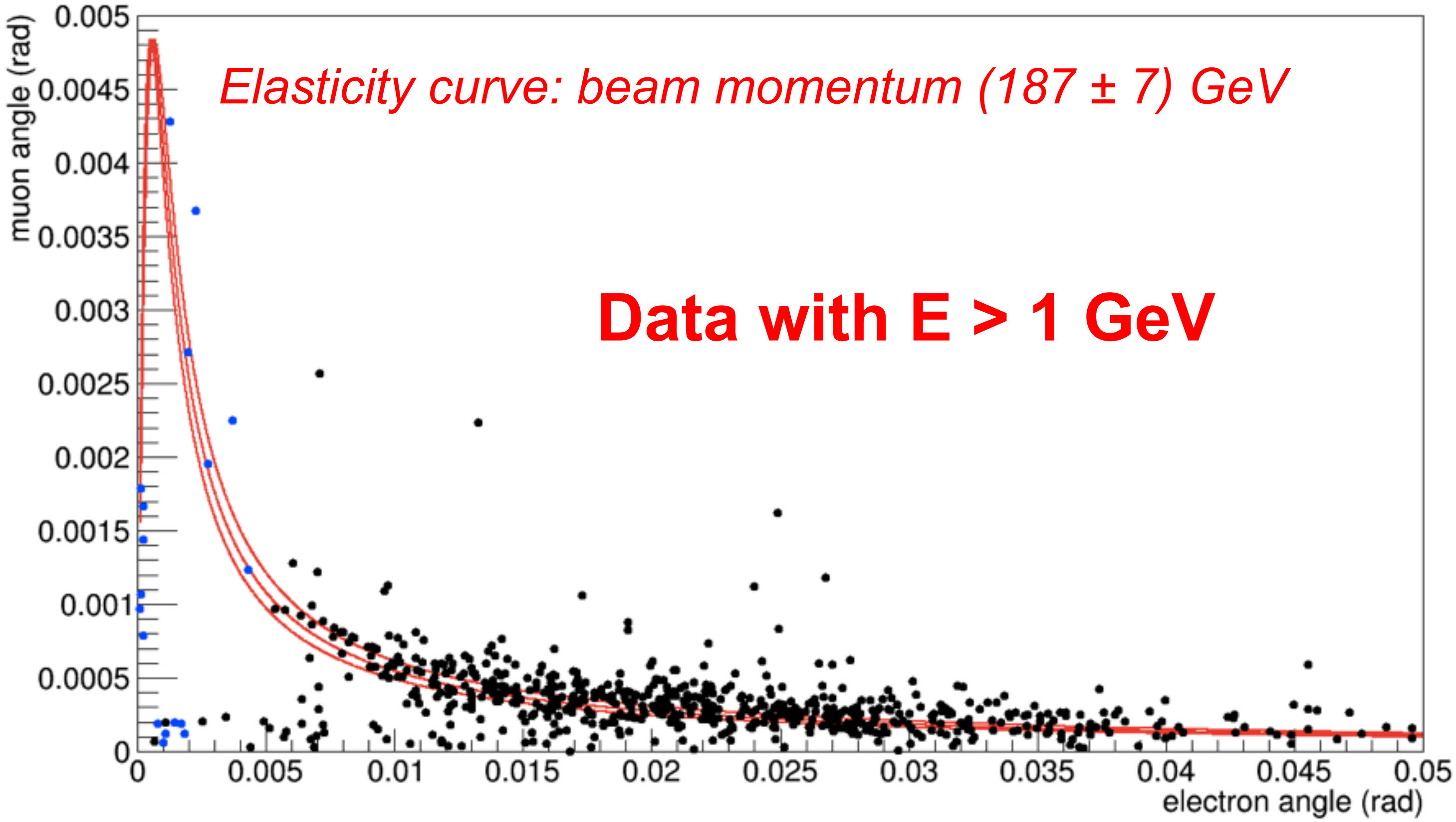
Simulation: Test beam 2018, GEANT4
Electron energy / angle correlation

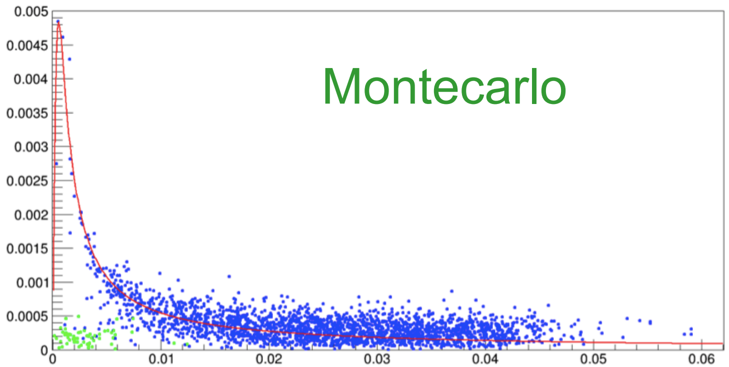
E_e *vs* θ_e

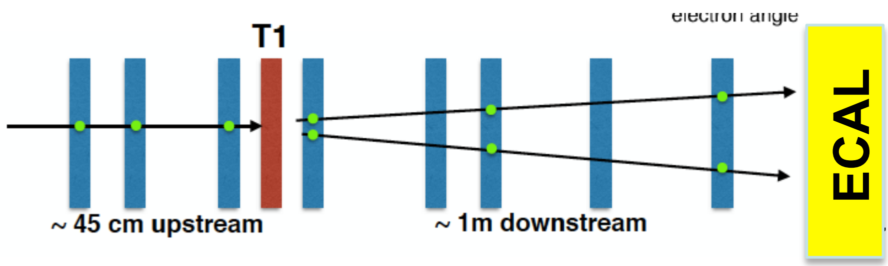


Testbeam in 2018

DATA using the Calorimeter



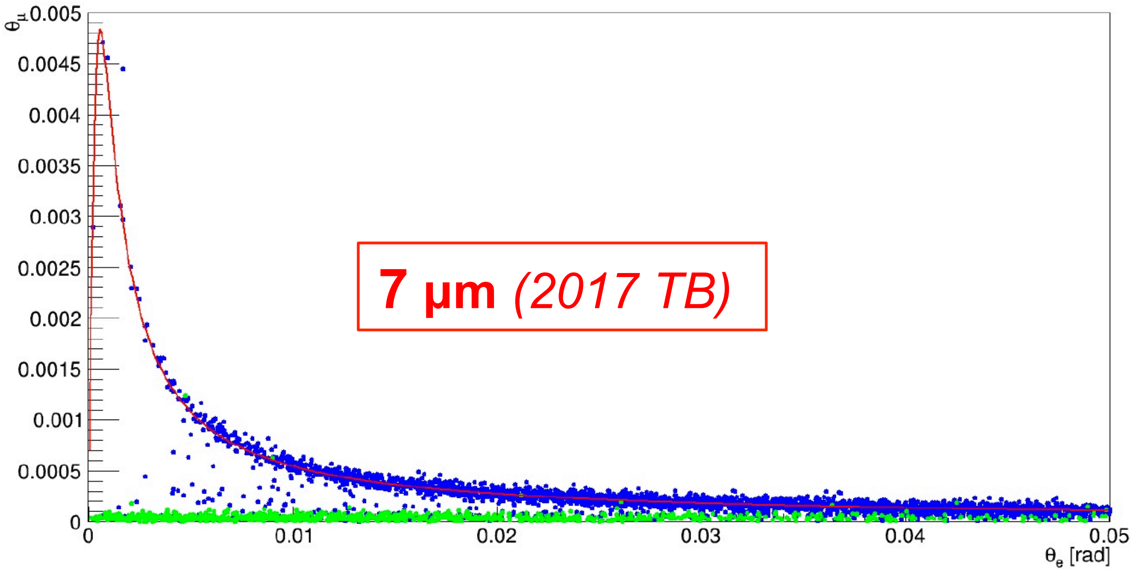




Testbeam in 2018

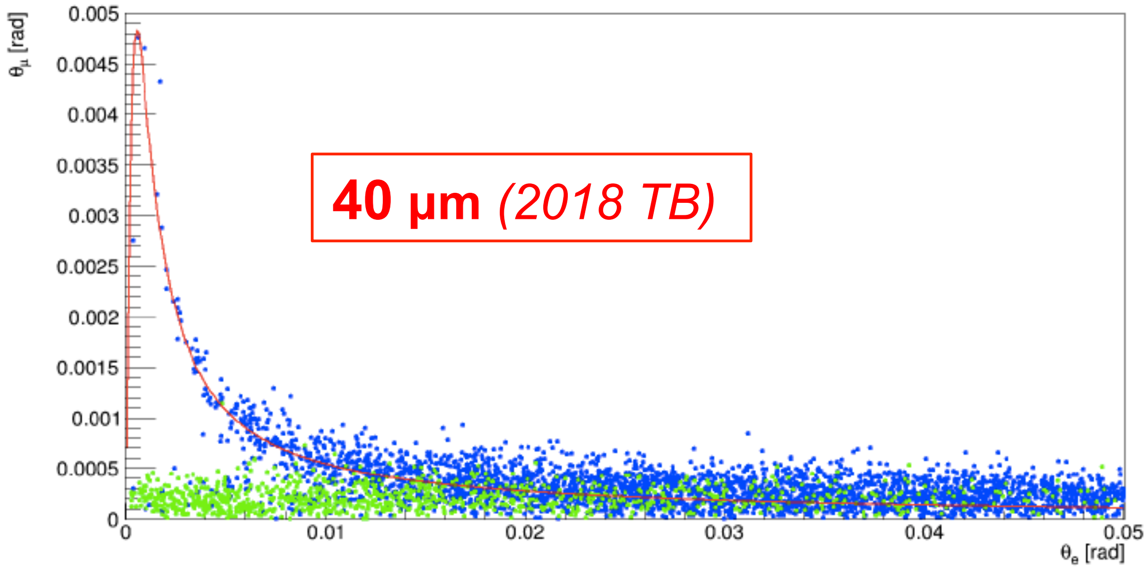
θ_μ *vs* θ_e correlation

Effect of the resolution with GEANT4



Signal *(elastic)*

Background *(pair production)*



Final detector MUST have the best resolution possible to be achieved with a Si tracker

Testbeam in 2018

Ongoing studies (data still being taken):

- Investigate the Efficiency of the selection
  - tracking algorithms
  - Use of coplanarity
  - Elasticity: d= d(P,γ) of the angles from the elasticit curve.
  - Common vertex constraint of the tracks at the target (muon in, muon out, electron candidate)
  - Cut in electron energy

DATA will be taken until 12^{th} november 2018

Possible location of μONe at CERN M2

- ***Between BSM and COMPASS***

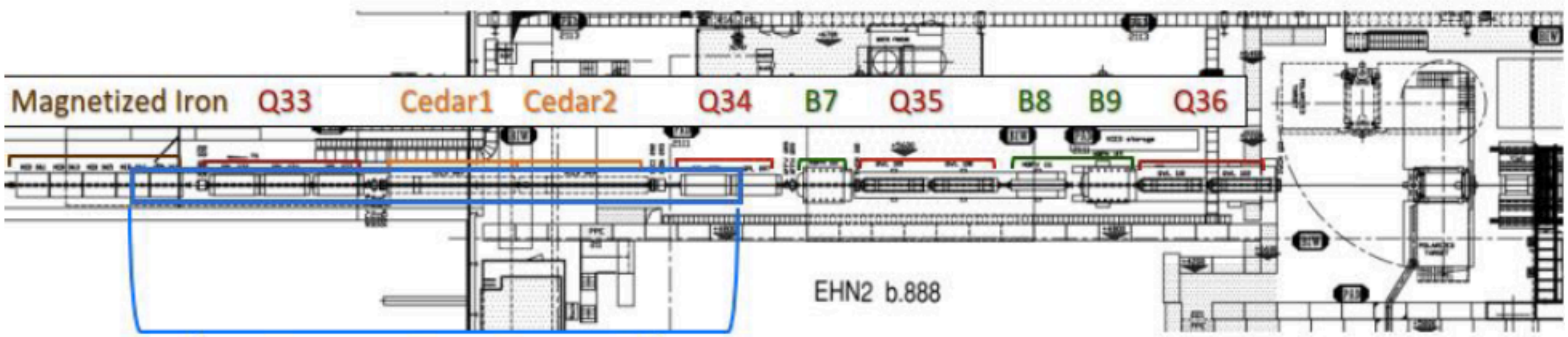
1/ μ-e setup upstream of present COMPASS experiment, i.e. within M2 beam-line

- More upstream of Entrance Area of EHN2 (Proposed by Johannes B. & Dipanwita B.)

- Pro: Could allow running μ-e/μ-p_{Radius} in parallel.

- Questions: will require displacements (also removal) of some M2 components.

- Beam(s) compatibility for μ-e & μ-p_{Radius} : Optic's wise looks OK (see Add. Sl.14 from D.B.)



μ-e Upstream (under-study) ~ 30 m ⇐ Σ length required (updated): 35 m targets + trackers + 0.8 m Ecals + 3 m (μ-Filter) ~ 39 m

10

(Studies by J. Bernhard and D. Banerjee)

Tools to do the measurement

Theory

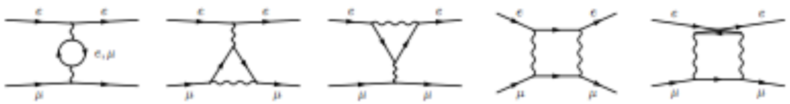
(From M. Passera)

Our final TH goal: a running MC for the ratio of the SM cross sections in the signal and normalization regions below, at the level, of 10ppm

Muon-electron scattering: theory progress

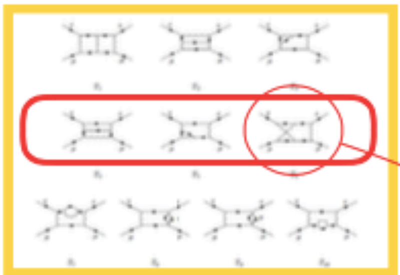
μe

- NLO QED corrections known & checked. MC @ NLO ready and tailored to the fixed target kinematics.



Pavia Group

- NNLO: Missing MI for the planar 2-loop box diagrams computed.

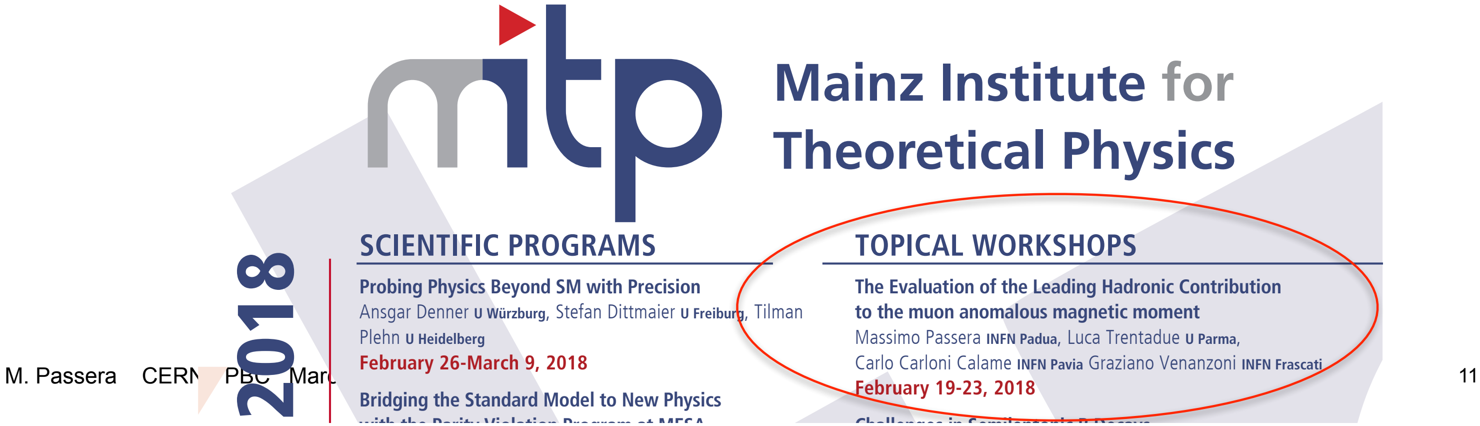


Mastrolia, MP, Primo & Schubert, arXiv:1709.07435.

Non-planar: not yet!

- NNLO amplitudes: virtual 2-loop, real-virtual, double real, automation, subtractions... — Mastrolia, Ossola, MP, Primo, Schubert, Torres
- NNLO hadronic contributions — Fael, MP
- Fixed-order NNLO + Resummation — Broggio, Signer, Ulrich
- Towards a MC at NNLO — Pavia group, Czyz
- Interplay with lattice calculations — Marinković

2nd MUonE theory workshop: Mainz - Feb 2018



next *hands-on* **workstop/thinkstart** strongly focussed on NNLO & theory MCs

Physik-Institut, University of Zurich, February 2019

Tentative timeline of the project:

Studies with GEANT4 for:

optimize geometrical configuration (target modularity, material budget)
number and thickness of Si layers
trigger studies
calorimetry (trasversal size, geometrical configuration,…)

Plans to have Beam Tests in 2019-2020 with electrons

to finalize Multiple Scattering studies

Detector elements:

under scrutiny now existing solutions for sensors and electronics/DAQ from CMS upgrade, (LHCb upgrade, BELLE II?)

Start taking data in 2021 with M2 beam (approved for COMPASS running) with part of the detector

Summary

We propose to measure $\mu e \rightarrow \mu e$ scattering in the space-like region with the existing CERN North Area μ beam and a detector which should not require R&D for new technologies

We are convinced that the physics case is extremely important (and timely!). The experiment is very challenging from the experimental point of view considering the systematic uncertainty which must be achieved, and hopefully it is doable in a relative short timescale

Quote from F. Jegerlehner (MITP workshop, Mainz 21-23 feb 2018)

- This experiment G. Abbiendi et al. is absolutely important also as it allows for for direct crosschecks with lattice QCD results and is has completely different systematics. Even a 5% crosscheck would be very helpful to scrutinize the HVP issue, and last bu not least whether the observed deviation is a real BSM effect.

Spares

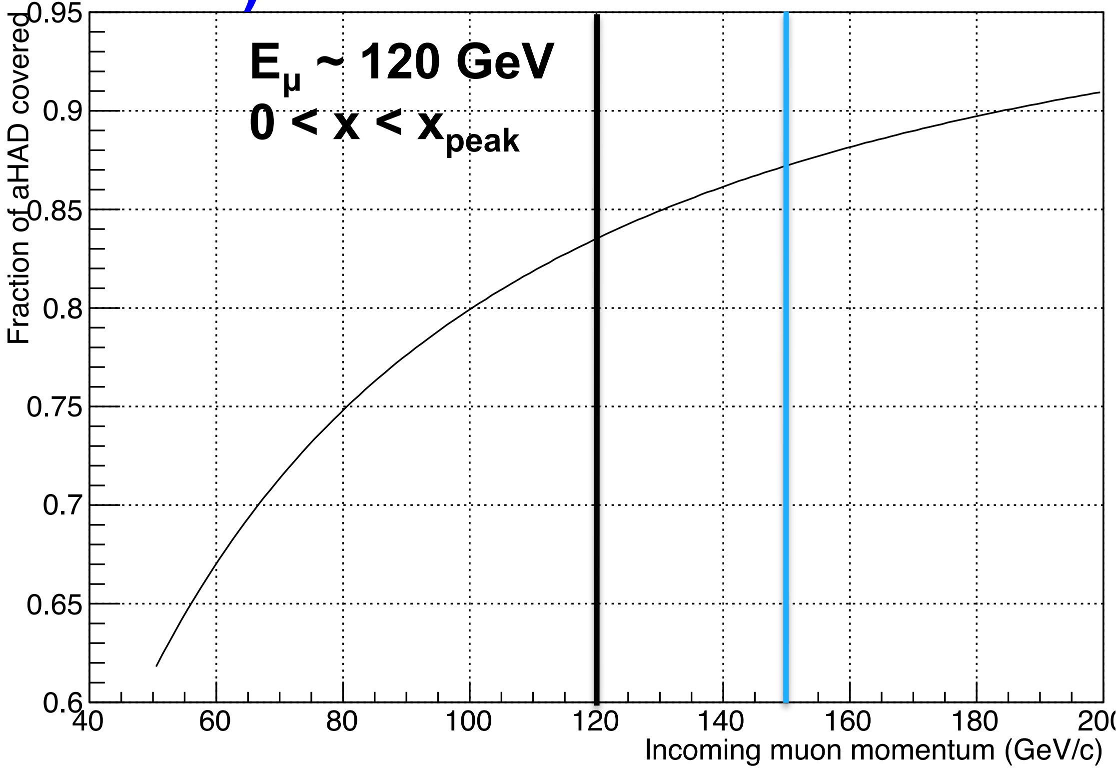
Measure the Hadronic Leading Order contribution (HLO) to the muon g-2 in the space-like region

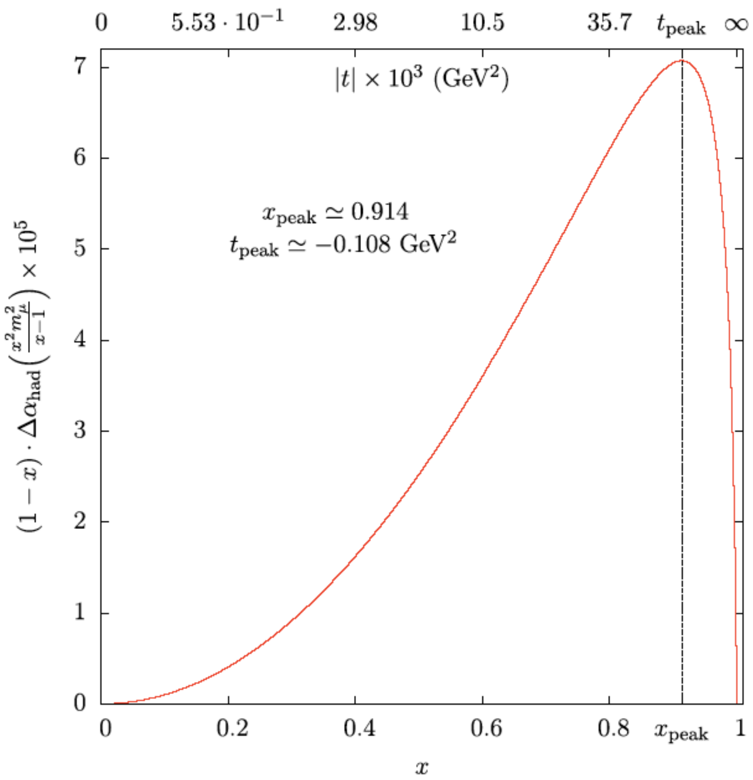
Proposed by:
G. Abbiendi, C.M. Carloni Calame, U. Marconi, C. Matteuzzi, G.Montagna, O. Nicrosini, M. Passera, F. Piccinini, R. Tenchini, L.Trentadue, G.Venanzoni

Reference:
G. Abbiendi et al., Eur. Phys. J. C (2017) 77:139. doi :10.1140/epjc/s10052-017-4633-z.

Optimal Muon Beam Momentum

Fraction of the a_μ^{HLO} integral as a function of the muon beam momentum: p_μ = 150 GeV → 87% of the integral (0 < x <0.93).





Beyond the kinematic limit the integral can be determined using pQCD & time-like data, and/or lattice QCD results.

Activity on the theory side

1. QED NLO corrections. Easy.

2. Resummation of dominant corrections up to all orders, matched with NLO corrections. Non-trivial issue: mass effects in this case are important

3. NNLO corrections: some classes of NNLO re-usable from existing Bhabha calculations, some new due to different mass scales ($\mathbf{m}_\mu$ and $\mathbf{m}_e$). In any case, NNLO must be matched with 1. and 2. [references: Eur. Phys. J. C 66 (2010) 585 and references therein]

4. Development of dedicated MC tools including all the above ingredients

5. Detailed study of all the mentioned corrections, comparison among independent calculations, estimate of further-missing higher-order corrections

Physics motivations

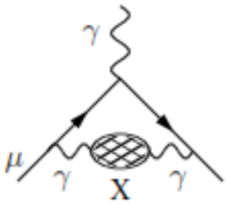
From F. Piccinini La Thuile 2018

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm HLO} + a_\mu^{\rm HHO}$$

- QED perturbative corrections known up to 4 loops plus 5 loops partial calculation:
  $a_\mu^{\rm QED} = 116584718.86(30) \times 10^{-11}$ $\sim$ 99.99% of the total

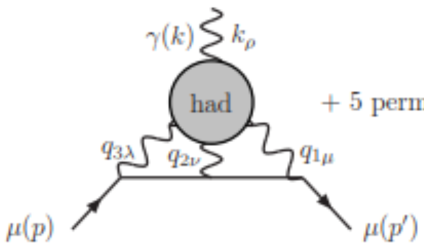
  T. Aoyama, M. Hayakawa, T. Kinoshita; S. Laporta, E. Remiddi; M. Passera
- $a_\mu^{\rm HLO} = 6894.6(32.5) \times 10^{-11} \Longrightarrow$ largest source of uncertainty

  F. Jegerlehner, MITP Workshop, 19-23 February 2018, Mainz



- Hadronic light-by-light: $a_\mu^{\rm LxL} = 103.4(28.8) \times 10^{-11}$

  F. Jegerlehner, MITP Workshop, 19-23 February 2018, Mainz



+ 5 permutations of the q_i

- Hadronic HO vacuum polarization: $a_\mu^{\rm HHO} = -87.0(0.6) \times 10^{-11}$
- two loop electroweak radiative corrections: $a_\mu^{\rm EW} = 153.6(1.1) \times 10^{-11}$

  Gnendiger, Stöckinger, Stöckinger-Kim