PHY127 Formula Sheet

Mechanics

Velocity	$\vec{v} = \frac{d\vec{r}}{dt}$
Acceleration	$\vec{a} = \frac{dv}{dt}$
Relativity	$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
	$t = \gamma t_0 \qquad x = \frac{1}{\gamma}\, x_0 \qquad p = \gamma\, m\, v$
Newtons second law	$\sum \vec{F} = m\,\vec{a}$
Newtons second law of rotation	$\sum \tau = I\alpha$
Angular acceleration	$\alpha = d\omega/dt$
Kinetic energy	$K = \frac{1}{2} m v^2$
Centripetal force	$F_z = \frac{m\,v^2}{r}$
Angular velocity	$\omega = \frac{d\varphi}{dt} = \frac{v}{r}$
Angular momentum	$\vec{L} = \vec{r} \times \vec{p}$
Drag force	$\vec{F}_D = -b\,\vec{v}$ (fluids $b = 6\,\pi\,\eta\,R$)
Torque	$\vec{\tau} = \vec{r} \times \vec{F}$
	$\vec{\tau} = \frac{dL}{dt}$
Impulse	$\vec{F}\Delta t = \Delta\vec{p} = m\Delta\vec{v} + \Delta m\vec{v}$

r, R	radius	m	mass	η	viscosity	b	drag constant
t	time	γ	Lorentz factor	p	momentum	I	moment of inertia

Electrodynamics

E-field of point charge	$\vec{E} = k\,\frac{Q}{r^2}\,\hat{r}$
Coulomb force	$\vec{F}_C = q\,\vec{E} = k\,\frac{Q}{r^2}\,\hat{r}$
Potential difference	$V_{AB} = -\int_A^B \vec{E} d\vec{s}$
Current	$I = \frac{dQ}{dt}$
Electric Power	$P = V\,I$
Lorentz force	$\vec{F}_L = q\,(\vec{v} \times \vec{B})$ (relativistic: $\vec{F}_L = q\,\vec{E} + q\,(\vec{v} \times \vec{B})$)
Power	$P =$ energy/time
Ohm's law	$V = R\,I$

q, Q	charge	k	coulomb constant	r	radial distance
v	velocity	B	magnetic field	R	resistance
m	mass	ϵ_0	el. permittivity (vacc.)	μ_0	mag. permeability (vacc.)

Oscillations and Waves

relations wavelength frequency	$k = \frac{2\pi}{\lambda} \quad \omega = 2\,\pi\,\nu \quad v = \lambda\,\nu$
Wave equation	$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$
	traveling wave $y(x,t) = A\,sin(k\,x \pm \omega\,t)$
	standing waves $y(x,t) = 2\,A\cos\omega\,t\sin k\,x$
Radiation Power	$P_S = \epsilon\,\sigma\,A\,(T^4 - T_0^4)$
Black-body radiation	$\lambda_{max} = 2.898\,\frac{\text{mm K}}{T}$
Intensity light	$I = \frac{P}{Area} = c\,\epsilon_0\,E^2$
Refraction (Snellius)	$\frac{\sin(\alpha)}{\sin(\gamma)} = \frac{n_2}{n_1}$
Diffraction	at the slit $\sin(\theta_n) = \frac{n\,\lambda}{d}$ - minima
	at the grid $\sin(\theta_n) = \frac{n\,\lambda}{d}$ - maxima
	at the hole $\theta_{1/2} \approx 1.22\,\frac{\lambda}{d}$ 1^{st} minimum
	at rings $D = n\,\lambda\frac{2\,L}{d}$

A	amplitude	λ	wavelength	ν	frequency	k	wave number
ϵ_0	el. permittivity	c	speed of light	φ	phase	E	electric field
ω	angular frequency	d, D	distance	θ	angle	T	temperature

Quantum mecanics

Uncertainty Principle	$\Delta x\Delta p \geq \frac{h}{4\,\pi} = \frac{1}{2}\hbar \quad \Delta E\Delta t \geq \frac{1}{2}\hbar$
Bragg angle	$\sin(\theta_{B,n}) = \frac{n\lambda}{2d}$
Energy of a photon	$E = h\,\nu$
Momentum of a photon	$p = \frac{h}{\lambda}$
Magnetic Moment	$\vec{\mu} = \frac{g\,e}{2\,m}\,\vec{L}$
Torque	$\vec{\tau} = \vec{\mu} \times \vec{B}$
Potential Energy	$U = -\vec{\mu} \cdot \vec{B}$
Precession frequency	$\omega_p = \frac{\mu B}{L}$
Larmor frequency	$\omega_L = \gamma\,B_z$
Rydberg Formula	$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{m^2} - \frac{1}{n_i^2}\right)$ (m, n shell number)
Work function	$h\,\nu - \phi = e\,V_0 = (\frac{1}{2} m\,v^2)_{max}$
de Broglie	$\lambda = \frac{h}{p}$
Energy of particle	$E = \frac{p^2}{2\,m} + U$
Energy of particle with mass	$E^2 = (mc^2)^2 + (cp)^2$
Momentum relativistic particle	$p \approx \frac{E}{c}$

x	position	m	mass	B	magnetic field
ν	frequency	c	speed of light	γ	gyromagnetic ratio
V	voltage	ϕ	electrostatic potential	e	electron charge
h	Plancks constant	L	angular momentum	g	Landé-factor

Schroedinger equation (time independent)

1 Dimension	$-\frac{\hbar^2}{2\,m_p}\frac{\partial^2\Psi(x)}{\partial x^2}+U(x)\Psi(x)=E\,\Psi(x)$
	Solution $\Psi(x)=\sqrt{\frac{2}{d}}\sin\left(\frac{n\,\pi}{d}\,x\right)$
Particle in box	$E_n=\frac{h^2\,n^2}{8\,m_p\,d^2}$ with $n=1,2,3,\ldots$
3D Box	$E\Psi=-\frac{\hbar^2}{2\,m_p}\left(\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2}\right)+U(x,y,z)\Psi$
	Solution $E_{n_x,n_y,n_z}=\frac{\hbar^2\,\pi^2}{2\,m_p}\left(\frac{n_x^2}{d_x^2}+\frac{n_y^2}{d_y^2}+\frac{n_z^2}{d_z^2}\right)$
Sphere	$E\Psi=-\frac{\hbar^2}{2\,m}\left[\frac{\partial}{\partial r}r^2\frac{\partial\Psi}{\partial r}-\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2\Psi}{\partial\phi^2}\right]+U\Psi$
	Solution $\Psi(r,\theta,\phi)=\Psi_r(r)\,\Psi_\theta(\theta)\,\Psi_\phi(\phi)$
Hydrogen atom	$E_0=\frac{k^2e^4\,m_p}{2\,\hbar^2}\approx 13.6\text{eV}$ $E_n=-\frac{Z^2}{n^2}\,E_0$
	$L_z=m\,\hbar$ $L=\hbar\sqrt{(l(l+1))}$

E_i	energy level	l,n,m	quantum number	k	wavenumber	m_p	mass
Z	atomic number	L	angular momentum	d	length, width		
Ψ	wave function	θ	azimuthal angle	ϕ	polar angle	r	radius

Radiation

Notation elements	Nuclei ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{Y}_{\mathrm{N}}$ A = Z + N
Radioactive decay	$N(t)=N_0\,e^{-\lambda\,t}=N_0\,e^{-\frac{t}{\tau}}$ $\tau_{\frac{1}{2}}=\ln(2)\,\tau=\frac{\ln(2)}{\lambda}$
Radioactive equilibrium	$\frac{N_A}{N_B}=\frac{\lambda_B}{\lambda_A}$
Equivalent dose	D_{eq} [Sv] $=\omega_R\,\omega_T\,D_{abs}$ [Gy]
1 Gy = 1 $\frac{\mathrm{J}}{\mathrm{kg}}$	1 R = 0.01 Gy = $2.58\cdot10^{-4}\,\frac{\mathrm{C}}{\mathrm{kg}}=2.08\cdot10^{9}\,\frac{\text{ions pairs}}{\mathrm{cm}^3}$
Attenuation of X-rays	$I(x)=I_0\,e^{-\mu\,x}$ or $I(x)=I_0\,e^{-\mu\,\rho\,x}$
	$I(x)=I_0\,e^{-\sum_{i=1}^{N}\mu_i\,x}$ for $i=1$ to N
Compton effect	$\Delta\lambda=\lambda'-\lambda=\frac{h}{m_e\,c}(1-\cos\theta)$ (here λ, λ': wavelength)
Nuclear binding energy	$E_B=Z(m_p\,c^2)+N(m_n\,c^2)-M\,c^2$ (M mass nucleus)

A	nucleons	τ	mean lifetime	M	mass nucleu
N	neutron	μ	attenuation coefficient	m_p	mass proton
Z	atomic number	λ	decay constant	m_n	mass neutron
I, I_0	intensity	ρ	density	m_e	mass electron
N	number of atoms	ω_R, ω_T	weighting factors	D_{xy}	absorbed dose

Units and Constants

Units

Force	1 Newton (N) = 1 $\frac{\mathrm{kg\cdot m}}{\mathrm{s}^2}$
Energy	1 Joule (J) = 1 N m = 1 $\frac{\mathrm{kg\cdot m}^2}{\mathrm{s}^2}$
Power	1 Watt (W) = 1 $\frac{\mathrm{J}}{\mathrm{s}}$ = 1 $\frac{\mathrm{kg\cdot m}^2}{\mathrm{s}^3}$
Pressure/stress	1 Pascal (P) = 1 $\frac{\mathrm{N}}{\mathrm{m}^2}$ = 1 $\frac{\mathrm{kg}}{\mathrm{m\cdot s}^2}$ (1 bar = 10^5 Pa)
Charge	1 Coulomb (C) = 1 A s
Electric field	1 $\frac{\mathrm{N}}{\mathrm{C}}$ = 1 $\frac{\mathrm{V}}{\mathrm{m}}$
Resistance	1 Ohm (Ω) = 1 $\frac{\mathrm{V}}{\mathrm{A}}$
Electrical capacitance	1 Farad (F) = $\frac{\mathrm{C}}{\mathrm{V}}=\frac{\mathrm{J}}{\mathrm{C}^2}$

Constants

Gravitational acceleration	$g=9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}$
Boltzmann constant	$k_B=1.381\cdot10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$
Absolute zero	$T_0=0$ K $=-273.15$ °C
Universal gas constant	$R=8.31\,\frac{\mathrm{J}}{\mathrm{mol\,K}}$
Avogadro constant	$N_A=\frac{R}{k_B}=6.022\cdot10^{23}\,\frac{1}{\mathrm{mol}}$
Elementary charge	$e=1.6\cdot10^{-19}$ C
Electrical discharge in air	$E_{max}=3\cdot10^6$ V/m
Coulomb constant	$k=\frac{1}{4\pi\epsilon_0}$
Electric field constant	ϵ_0=$8.85\cdot10^{-12}\,\frac{A\ s}{V\ m}$
Magnetic field constant	μ_0 =$4\pi\cdot10^{-7}\ V\ s\ A^{-1}\ m^{-1}$
Mass of the electron	m_e =$9.11\cdot10^{-31}$ kg
Mass of the proton	m_p =$1.67\cdot10^{-27}$ kg
Electron volt	1 eV =$1.602\cdot10^{-19}$ J
Planck's constant	h =$6.63\cdot10^{-34}$ Js
	$\hbar=\frac{h}{2\pi}$
Stefan's constant	σ =$5.67\cdot10^{-8}\,\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{K}^4}$
Rydberg constant for hydrogen	R_{H} =$1.09678\cdot10^{7}$ m^{-1}
Bohr magneton	$\mu_B=\frac{e\hbar}{2\,m_e}=9.274\cdot10^{-24}\,\frac{\mathrm{J}}{\mathrm{T}}$

Calculation rules

Geometry

Position vector	$\vec{r}_i = (x_i, y_i, z_i)$
Dot product	$\vec{r}_1 \cdot \vec{r}_2 = x_1x_2 + y_1y_2 + z_1z_2 = \|\vec{r}_1\|\|\vec{r}_2\| \cos\theta$
Vector product	$\vec{r}_1 \times \vec{r}_2 = (y_1z_2 - y_2z_1, z_1x_2 - z_2x_1, x_1y_2 - x_2y_1) =$
	$\|\vec{r}_1\|\|\vec{r}_2\| \sin\theta\, (\hat{r}_1 \times \hat{r}_2)$
	$\hat{x} \times \hat{y} = \hat{z}$
Sphere	$A = 4\, r^2\, \pi \quad V = \frac{4}{3}\, r^3\, \pi$
Circle	$C = 2\, r\, \pi \quad A = r^2\, \pi$

Logarithm

$b^x = a \;\leftrightarrow\; \log_b(a) = x$

$e^x = a \;\leftrightarrow\; \ln(a) = x$

$\log_a xy = \log_a x + \log_a y$

$\log_a \frac{x}{y} = \log_a x - \log_a y$

$\log_a x^n = n \log_a x$

$\log_a b = \frac{\log_c b}{\log_c a}$

$\log_a b = \frac{1}{\log_b a}$

$\log_a 1 = 0$

$\log_a a = 1$

$\log_a a^x = x$

$\log_a \frac{1}{b} = -\log_a b$

$\log_{\frac{1}{a}} b = -\log_a b$

$\log_a b \cdot \log_b c = \log_a c$

$\log_{a^m} a^n = \frac{n}{m},\; m \neq 0$

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$\tan x = \frac{\sin x}{\cos x}$

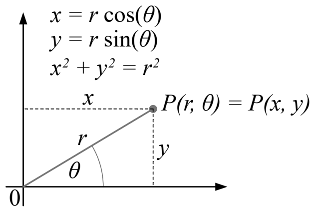
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\frac{\mathrm{d} \sin x}{\mathrm{d}x} = \cos x$ and $\frac{\mathrm{d} \cos x}{\mathrm{d}x} = -\sin x$



Periodic table

A **Y** **Z**

A: Nucleon no. of most frequent isotope
Y: Element
Z: Atomic number

1 **H** **1**																	4 **He** **2**
7 **Li** **3**	9 **Be** **4**											11 **B** **5**	12 **C** **6**	14 **N** **7**	16 **O** **8**	19 **F** **9**	20 **Ne** **10**
23 **Na** **11**	24 **Mg** **12**											27 **Al** **13**	28 **Si** **14**	31 **P** **15**	32 **S** **16**	35 **Cl** **17**	40 **Ar** **18**
39 **K** **19**	40 **Ca** **20**	45 **Sc** **21**	48 **Ti** **22**	51 **V** **23**	52 **Cr** **24**	55 **Mn** **25**	56 **Fe** **26**	59 **Co** **27**	57 **Ni** **28**	64 **Cu** **29**	65 **Zn** **30**	70 **Ga** **31**	73 **Ge** **32**	752 **As** **33**	79 **Se** **34**	80 **Br** **35**	84 **Kr** **36**
85 **Rb** **37**	88 **Sr** **38**	89 **Y** **39**	91 **Zr** **40**	93 **Nb** **41**	96 **Mo** **42**	98 **Tc** **43**	101 **Ru** **44**	103 **Rh** **45**	106 **Pd** **46**	108 **Ag** **47**	112 **Cd** **48**	115 **In** **49**	119 **Sn** **50**	122 **Sb** **51**	128 **Te** **52**	127 **I** **53**	131 **Xe** **54**
133 **Cs** **55**	137 **Ba** **56**		178 **Hf** **72**	181 **Ta** **73**	184 **W** **74**	1864 **Re** **75**	190 **Os** **76**	192 **Ir** **77**	195 **Pt** **78**	197 **Au** **79**	201 **Hg** **80**	204 **Tl** **81**	207 **Pb** **82**	209 **Bi** **83**	209 **Po** **84**	210 **At** **85**	222 **Rn** **86**
223 **Fr** **87**	226 **Ra** **88**																

139 **La** **57**	140 **Ce** **58**	141 **Pr** **59**	144 **Nd** **60**	145 **Pm** **61**	150 **Sm** **62**	152 **Eu** **63**	157 **Gd** **64**	159 **Tb** **65**	163 **Dy** **66**	165 **Ho** **67**	167 **Er** **68**	169 **Tm** **69**	173 **Yb** **70**	175 **Lu** **71**
227 **Ac** **89**	232 **Th** **90**	231 **Pa** **91**	238 **U** **92**	237 **Np** **93**	244 **Pu** **94**	243 **Am** **95**	247 **Cm** **96**	247 **Bk** **97**	251 **Cf** **98**	252 **Es** **99**	257 **Fm** **100**	258 **Md** **101**	259 **No** **102**	262 **Lr** **103**