Today:

torque
angular momentum
rotational inertia
precession

if time:

pressure
atmospheric pressure
fluids

PHY 117 HS2023

Week 4, Lecture 2
Oct. 11th, 2023
Prof. Ben Kilminster

Quiz 2:

	Unanswered	Right	Wrong
The spring constant would be the same on the moon than on the earth. ($k = \frac{mg}{\Delta x}$, and g is different on the moon)		answer 118	~~129~~

K is a constant, independent of the type of force.
If we pull on a spring with any force F, it will extend by Δx, so $K = \frac{F}{\Delta x}$.

More info...

$I\ \Delta x_m = \Delta x_{earth}/6$

$g_{earth} = 9.81\ m/s^2$

To calculate K, we hang a weight.

$$K = \frac{F}{\Delta x} = \frac{m g_{earth}}{\Delta x_{earth}}$$

$g_{moon} = 1.62\ \frac{m}{s^2}$

$g_{moon} = g_{earth}/6$

$$K = \frac{m g_{moon}}{\Delta x_{moon}} = \frac{m g_{earth}/6}{\Delta x_{earth}/6}$$

$$K = \frac{m g_{earth}}{\Delta x_{earth}}$$

yesterday

linear motion	rotational motion
$\bar{F} = m\bar{a}$	$\bar{\tau} = I\bar{\alpha}$

I is kind of like mass ($I = mr^2$) for one particle

$$\boxed{\text{Newton's second law of rotation} \quad \sum \bar{\tau} = I\bar{\alpha}}$$

I is a measure of how mass is distributed.

$$I = \sum m_i r_i^2$$

$$I = \int r^2\, dm$$

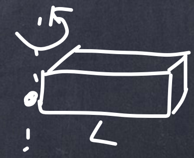
Examples of I, rotational inertia

Simple case: ring
total mass M, all at a radius R

rotating around center

$$I_{ring} = MR^2$$

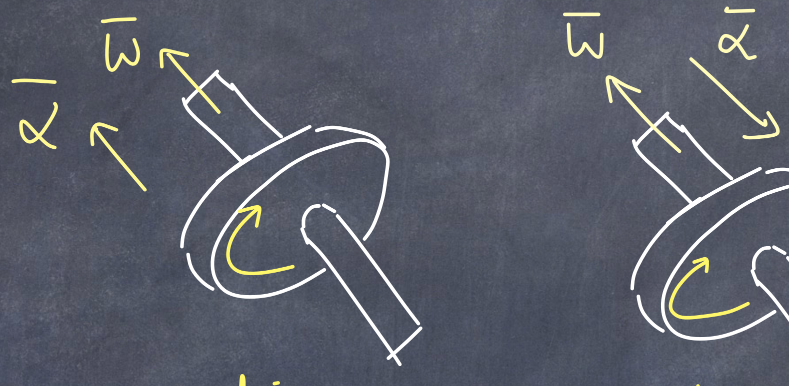
shape	I
* point	$I = MR^2$
* ring	$I = MR^2$
stick	$I = \frac{1}{3}ML^2$
sphere	$I = \frac{2}{5}MR^2$



In the script, I derive some of these.
In this class, you only need to know how to derive these (*)

$\bar{\tau} = I\bar{\alpha}$ what direction is $\bar{\alpha}$?

$\bar{\alpha}$ points toward $\bar{\tau}$



speeding up — slowing down

For $\bar{\alpha}$, the angular acceleration, it points in the same direction as $\bar{\omega}$ if $\bar{\omega}$ is getting bigger.

If $\bar{\omega}$ is slowing down, then $\bar{\alpha}$ points opposite to $\bar{\omega}$.

In linear motion, we have $\bar{p} = m\bar{v}$

and $\quad \sum \bar{F} = m\bar{a} = \frac{d\bar{p}}{dt} \quad (1)$

If $\sum \bar{F} = 0$, no net force, then $\frac{d\bar{p}}{dt} = 0$,

and momentum is conserved.

$$\underset{\text{initial}}{\bar{p}_i} = \underset{\text{final}}{\bar{p}_f}$$

In a rotating system $\bar{\tau} = \bar{r} \times \bar{F}$

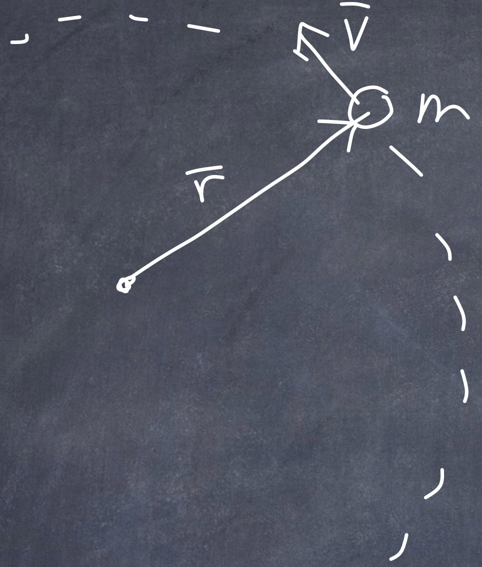
start with eq. (1), we take "$\bar{r} \times$" of both sides

$$\underbrace{\sum(\bar{r} \times \bar{F})}_{} = \frac{d(\bar{r} \times \bar{p})}{dt}$$

$$\sum \bar{\tau} = \frac{d\bar{L}}{dt}$$

where we define $\bar{L} \equiv \bar{r} \times \bar{p}$, which is the angular momentum.

Angular momentum: $\bar{L} = \bar{r} \times \bar{p}$



$$= \bar{r} \times (m\bar{v}) = m(\bar{r} \times \bar{v})$$

In a circle, $\bar{v} \perp \bar{r}$

$$\bar{r} \times \bar{v} = rv\sin\theta = rv(\sin 90^\circ)$$
$$= rv$$

So here, angular momentum for an object moving in a circle is $L = mvr$

Now we also know that $v = r\omega$

Therefore, $L = m(r\omega)r = \underbrace{mr^2}_{}\omega$
$$= I\omega$$

$$\boxed{\bar{L} = I\bar{\omega}}$$

angular momentum, inertia (rotational), angular velocity

analogy

$$\bar{p} = m\bar{v}$$

linear momentum, inertia (mass), linear velocity

If there are no external forces, then

$$\Sigma(r \times \bar{F}) = 0 = \Sigma \bar{\tau} = \frac{d\bar{L}}{dt}$$

If $\frac{d\bar{L}}{dt}$ is $\emptyset$, then $\bar{L}$ is constant.

$$\bar{L}_{before} = \bar{L}_{after}$$

conservation of angular momentum

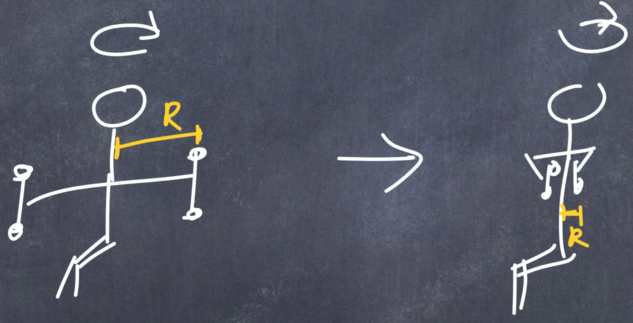
when there is no external torques

So $\vec{L}$ is conserved like $\vec{p}$

$$\boxed{\vec{L} = I\vec{\omega}}$$

this means that if we change I, then ω must also change because L stays constant.

$I = MR^2$ for the weights

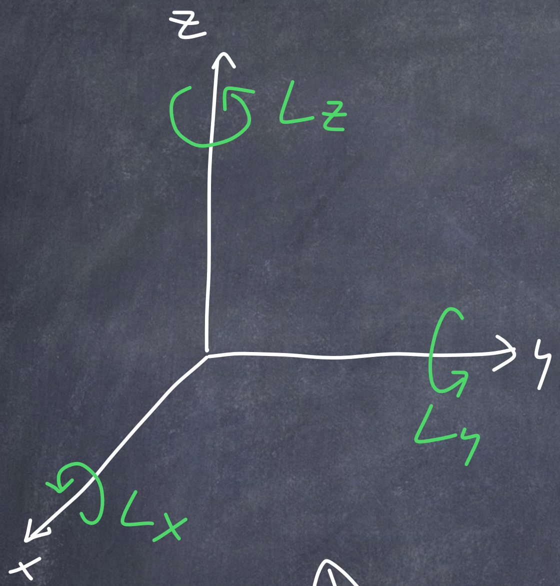


I is big → I is small

$$\underset{\substack{\uparrow \\ \text{same}}}{L} = \underset{\substack{\uparrow \\ \text{decrease}}}{I} \; \underset{\substack{\uparrow \\ \text{increase}}}{\omega}$$

Objects can spin around 3 axes.

So $\bar{L}$ can be



$$\bar{L} = L_x \hat{x} + L_y \hat{y} + L_z \hat{z}$$

Angular momentum must be conserved in all 3 directions, independently.

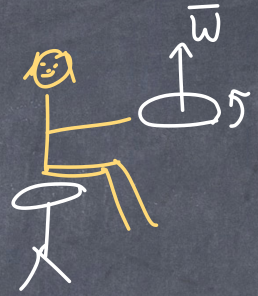
check your right-hand rule to see that the spin is consistent with the axis direction.

Initial :

chair can only rotate around the z axis

$\hat{z}$ ↑

Final:



wheel now has $\bar{L} = L_z \hat{z}$ $\underbrace{}$ rotating around the z axis.

In the direction, L_z is conserved.

Initial: Angular momentum around z-axis is zero.

$$L_z = 0$$

Finally : in z-direction

$$L_{z_{wheel}} + L_{z_{me}}$$

Because of conservation of $\bar{L}$,

$$\bar{L}_{initial} = \bar{L}_{final}$$

$$0 = L_{z_{wheel}} + L_{z_{me}}$$

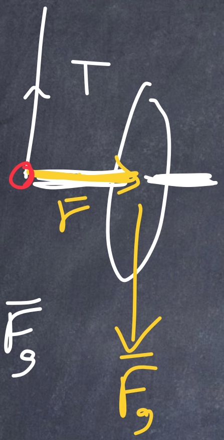
$$L_{z_{me}} = -L_{z_{wheel}}$$

precession

Remember that $\bar{\tau} = \frac{d\bar{L}}{dt}$

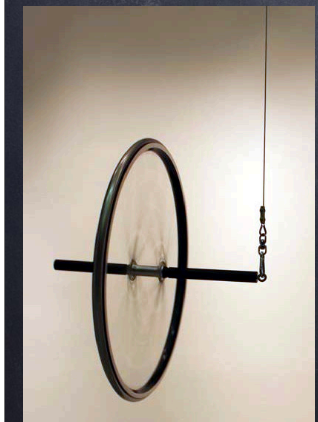
when not spinning, $\bar{\tau} = \bar{r} \times \bar{F} = rMg$

causes wheel to fall

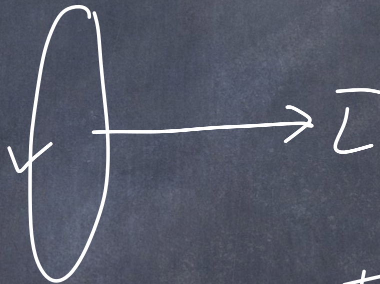


$\bar{\tau}$ ⊗

$\bar{T} = -\bar{F}_g$



But if we spin the wheel



Since $\bar{\tau} = \frac{d\bar{L}}{dt}$,

then $d\bar{L} = \bar{\tau}\, dt$

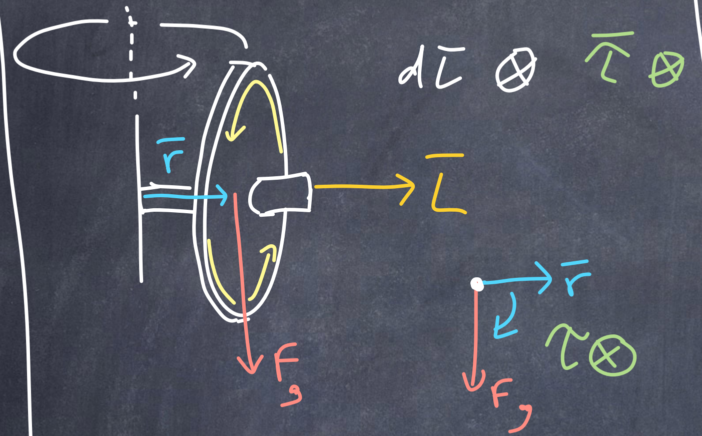
If we have a torque, then we change the angular momentum.

The direction of $d\bar{L}$ is the direction of $\bar{\tau}$.

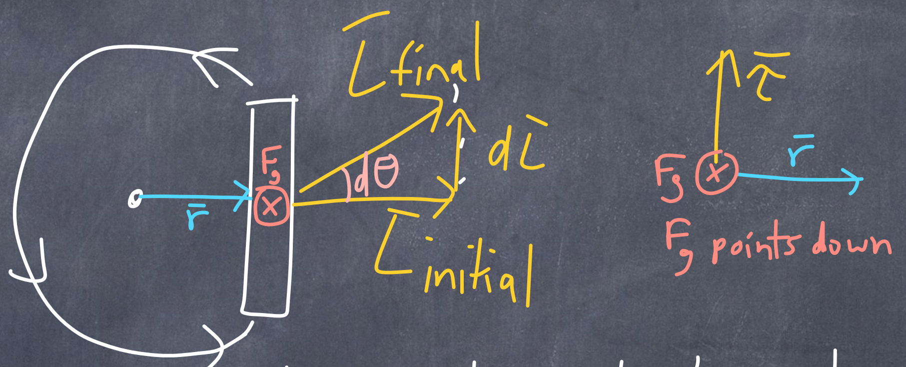
Here $\bar{\tau}$ ⊗

The amount of $d\bar{L} = \bar{\tau}\,dt = \underbrace{rMg}_{\tau}\,dt$ ①

view from side:



view from above: ②



we expect $\bar{L}$ to turn, to rotate in the direction of the torque.

from picture ②:

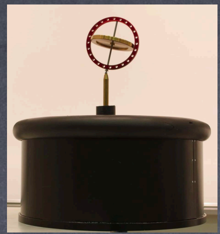
$$dL = L\,d\theta \qquad d\theta = \frac{dL}{L}$$

From ① $\quad d\theta = \dfrac{rMg\,dt}{L} \Rightarrow \underbrace{\dfrac{d\theta}{dt}}_{\omega} = \dfrac{rMg}{L}$

ω_p : angular velocity of precession

$$\omega_p = \frac{rmg}{L} = \frac{rmg}{I\omega} \leftarrow$$

ω : angular velocity of the spinning wheel

Angular momentum, torque, & precession will come up in NMR & MRI, (we will discuss in PHY 127)

New topic: Fluids

Fluids – what is pressure?

what is atmospheric pressure?

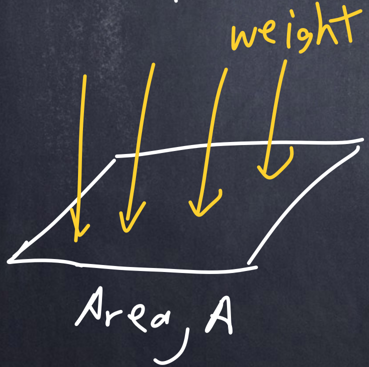
$$P = \text{pressure} = \frac{\text{force}}{\text{area}} \qquad \text{units } \left[\frac{N}{m^2}\right] \equiv [Pa]$$

pascal

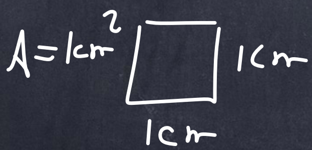
$$1\,Pa = \frac{1\,N}{m^2}$$

The atmospheric pressure at sea level is 101.325 kPa. This is the weight of all the air above us on some area.

weight ← force.



How much weight does the atmosphere feel like on an area of $1\,cm^2$

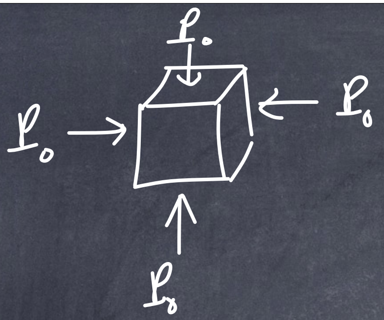


$$P_0 = 101325\,\frac{N}{m^2}$$

atm. pressure

$$F = P_0 A = 101325\,\frac{N}{m^2} \cdot 1\,cm^2 \cdot \frac{1\,m^2}{(100\,cm)^2}$$

$$F \cong 10\,N/cm^2 \Rightarrow m = 1\,kg$$



Pressure is not a vector.
Pressure pushes in all directions with the same value (at a given height)

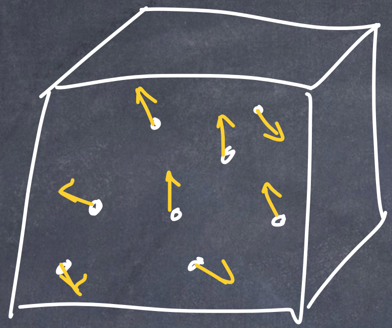
$A =$  $= \pi r^2 = 75\,cm^2$

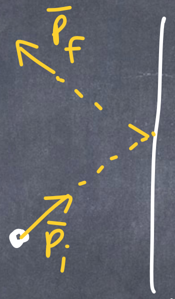
$F_{ATM} = P_0 \cdot A = 10 \frac{N}{cm^2} \cdot 75\,cm^2 = 750\,N$

$F_g = (m_{water})\, g = (0.5\,kg)\left(10 \frac{m}{s^2}\right) = 5\,N$

The force from the atmospheric pressure is much more than the weight due to gravity

where does pressure come from?





wall of a box

$\bar{p}_i$: initial momentum of molecule before it hits the wall

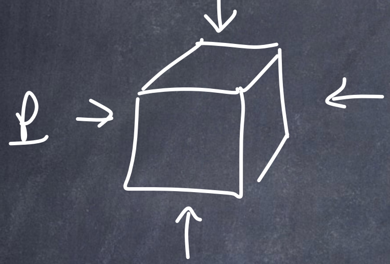
$\bar{p}_f$: final momentum

There is a change in momentum $\underbrace{\Delta \bar{p}}_{\text{impulse}} = \bar{p}_f - \bar{p}_i$

remember
$$\bar{F} = \underbrace{\frac{\Delta \bar{p}}{\Delta t}}_{p:\ \text{momentum}} = \underset{\uparrow\ \text{pressure}}{\underline{P}} \cdot \text{Area}$$

For N molecules,
$$\left. F = N \frac{\Delta p}{\Delta t} \right] \leftarrow \text{depends on velocity.}$$

If we apply a pressure to a substance of volume V, it gets compressed by some amount ΔV. (Note: ΔV is $(-)$)



We can define the Bulk modulus, B, to describe how much a substance compresses for a given pressure.

$$B \equiv \frac{-P}{\frac{\Delta V}{V}}$$

The $(-)$ sign here makes B $(+)$ because ΔV is $(-)$

material	B [Pa]
iron	100
lead	8
water	2

gases: B is very small (depends on temperature)

liquids + solids: B is large because it is hard to compress liquids and solids.