PHY117 HS2023

Week 7, Lecture 2
Nov. 1st, 2023
Prof. Ben Kilminster

Adiabatic expansion:

Gas volume expands without flow of heat in or out of the system.

$$\Delta U = \cancel{Q} - W \quad (Q \to 0)$$

$$\boxed{\Delta U = -W}$$

If system expands, system does work.
the work is then (+).

Then ΔU is (−) ⇒ decrease in internal energy
⟹ temperature decreases.

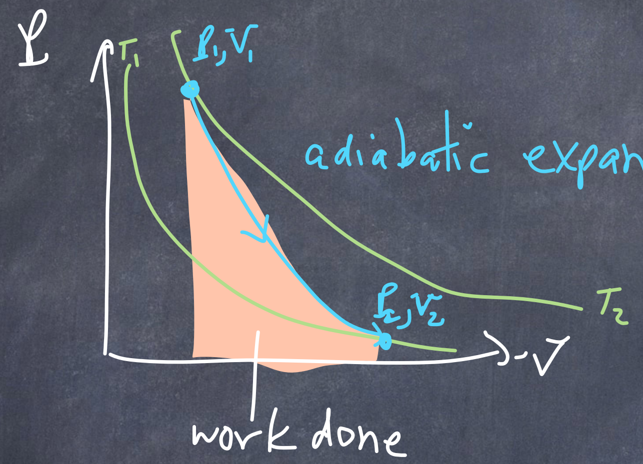
Adiabatic processes can happen in 2 ways:

1) so quick that heat can't be exchanged.
2) very slowly in a well-insulated system
"quasi-static adiabatic process"

Adiabatic process : $Q = 0$, $\Delta U = -W$

what is constant?

what is the work done?



we know that $dU = C_V dT$ ← for any ideal gas

$dW = P\,dV$ work done

$dU = -dW$ for an adiabatic process

For adiabatic processes:

$$\Delta U = -W$$

Adiabatic process with an ideal gas:

$$PV^{\gamma} = \text{constant}$$

where $\gamma = \frac{C_p}{C_v}$

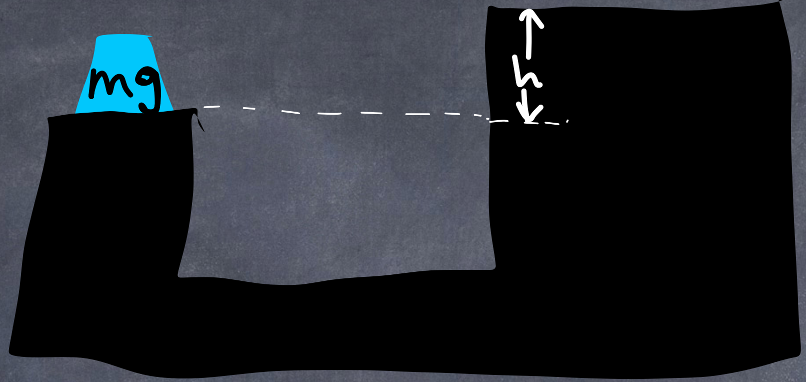
$$TV^{\gamma-1} = \text{constant}$$

$$W = \frac{1}{\gamma - 1}\left(P_1V_1 - P_2V_2\right)$$

work done by a gas expanding adiabatically

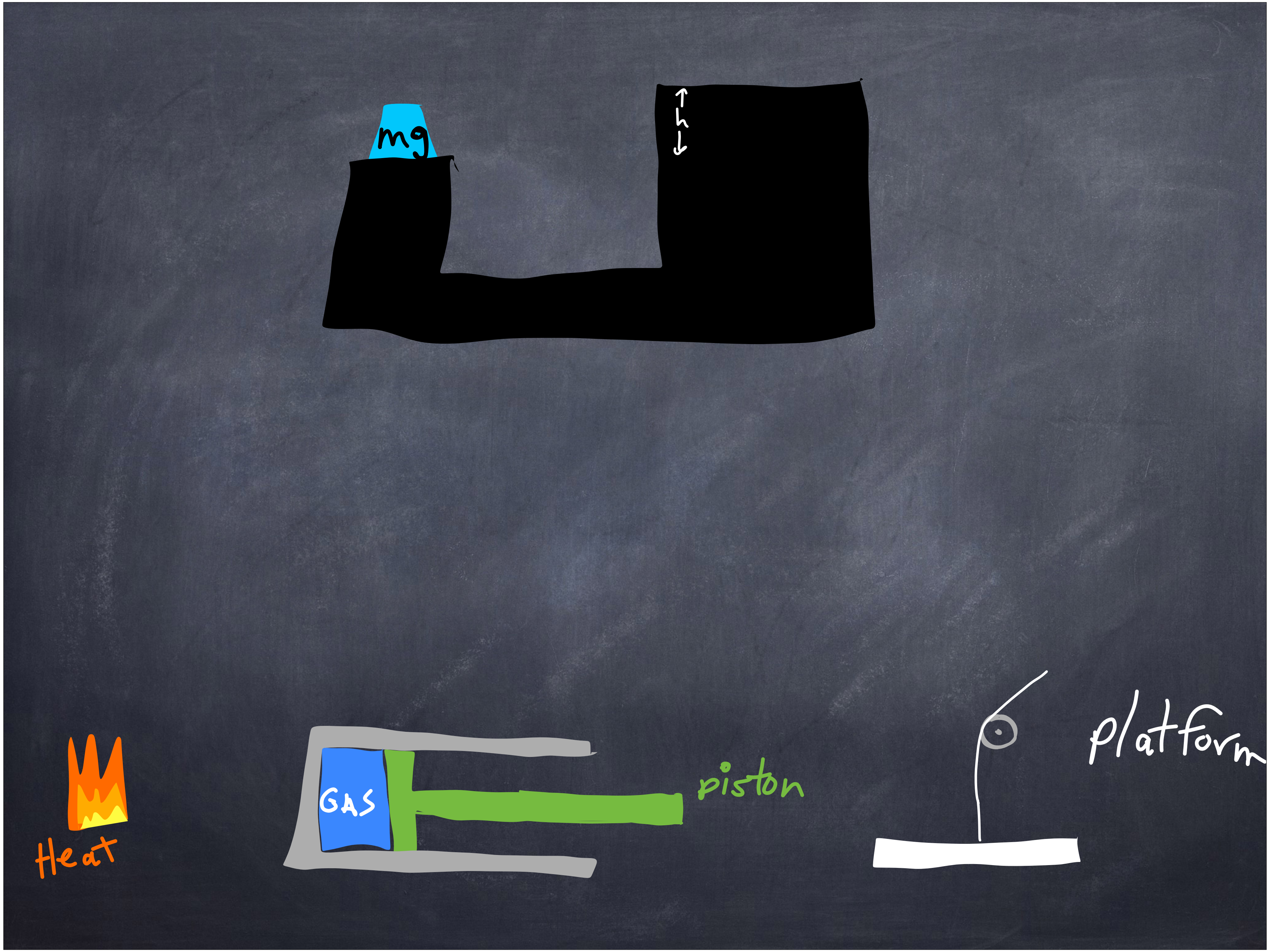
Derivations in script 2

suppose we want to lift this mass a height, h.

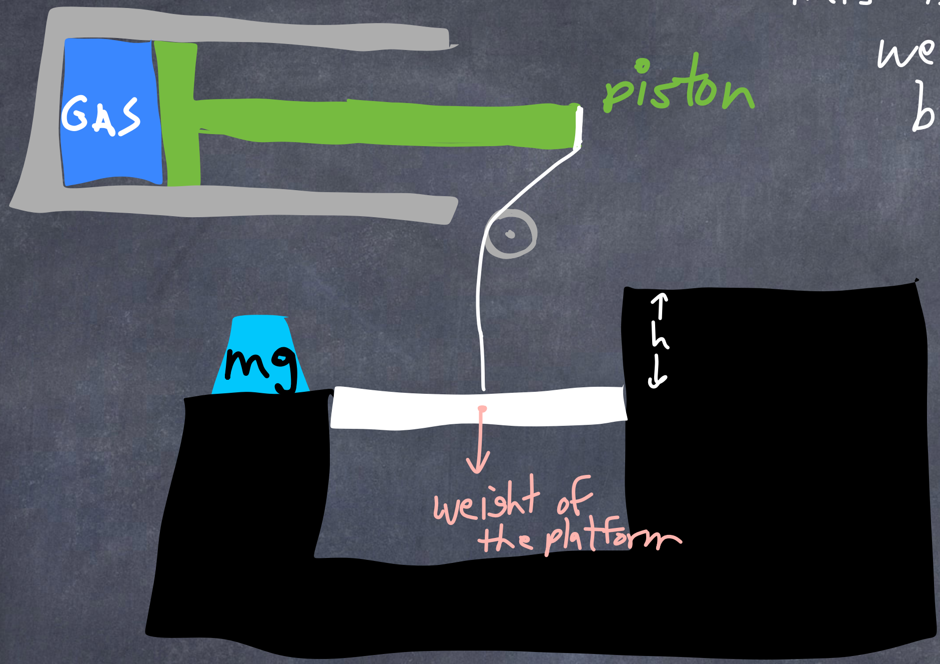


Requires work
$W = mgh$

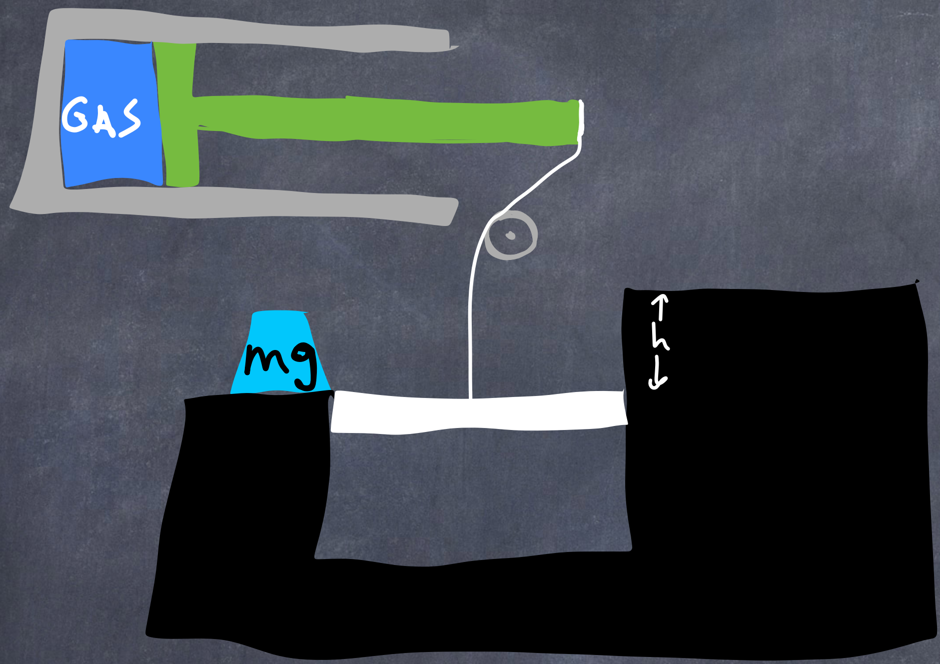
we want to use a gas to do this.

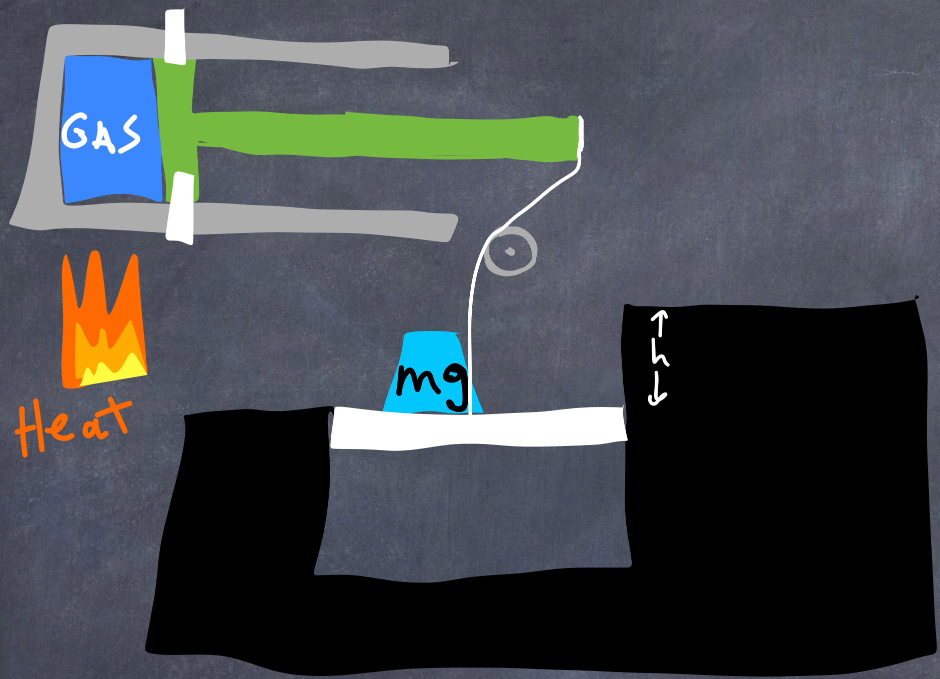

mg
h
Heat
GAS
piston
platform

Initially, gas is at P_1, V_1, T_1
This is at equilibrium, so weight of the platform balances the force from the gas pressure.



initial) At equilibrium, P_1, V_1, T_1



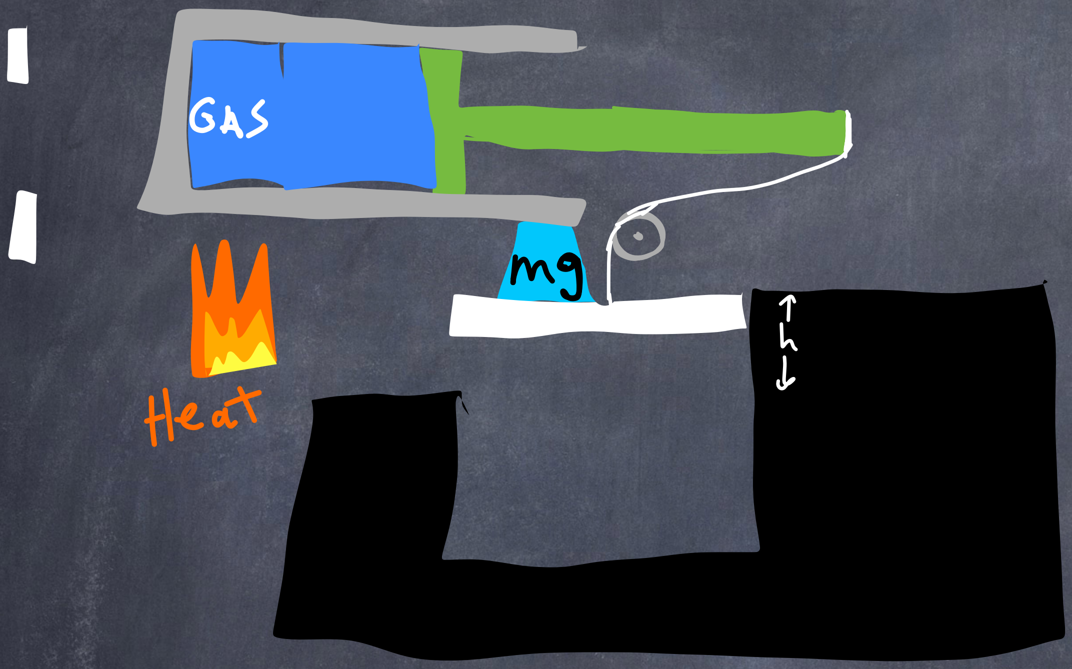


initial) At equilibrium, P_1, V_1, T_1

a) we fix volume to V_1, then heat gas at constant volume. So pressure increases to P_2. P_2 can now hold the weight mg. we slide weight onto platform

a) $P_1 \rightarrow P_2$, V_1 constant

b) $V_1 \rightarrow V_2$, P_2 constant



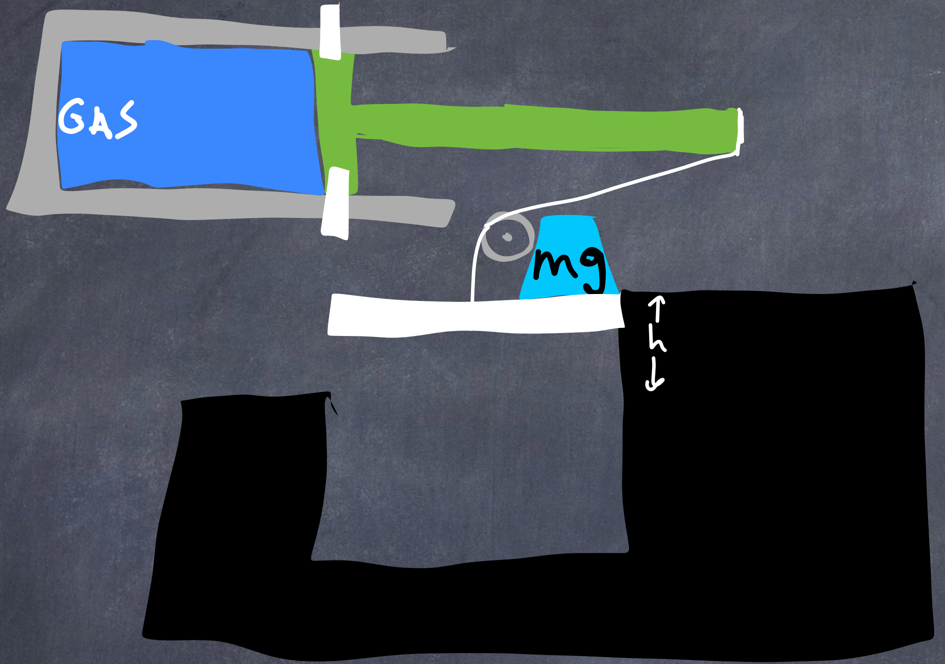
0) At equilibrium, P_1 V_1 T_1

a) We fix volume at V_1. Then heat gas at constant volume. So pressure increases to P_2. We slide weight on platform. The pressure P_2 can now hold the weight mg.

b) unfix the volume. Heat the gas until volume increases to V_2. This raises the weight a height, h.

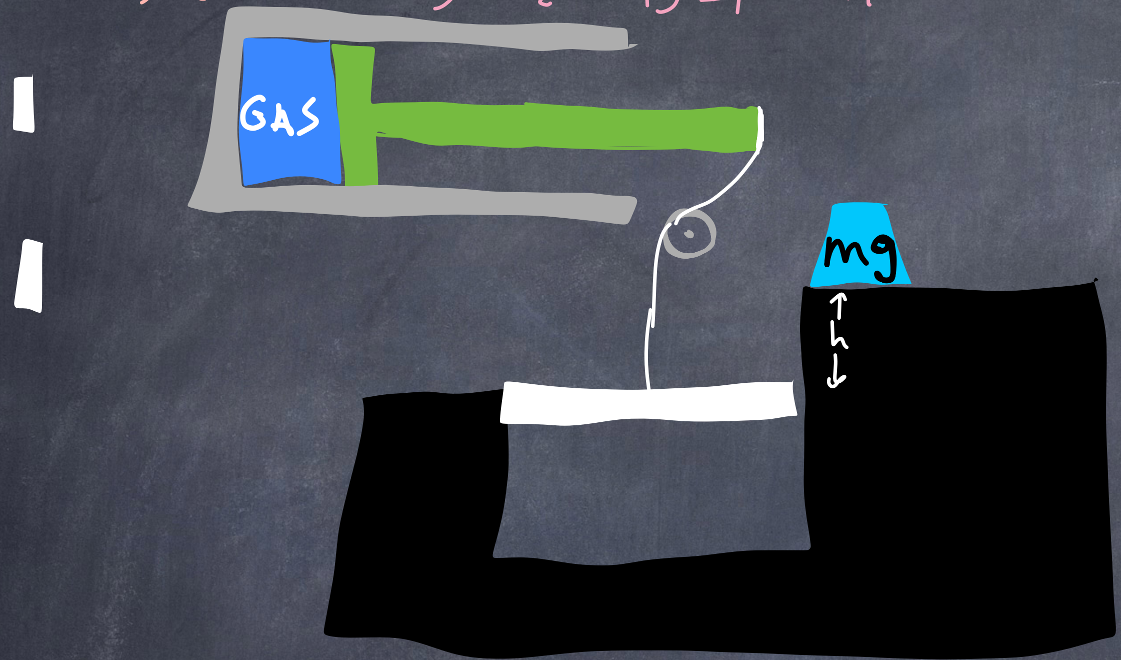
a) $P_1 \to P_2$, V_1 constant
b) $V_1 \to V_2$, P_2 constant
c) $P_2 \to P_1$, V_2 constant

0) At equilibrium, P_1, V_1, T_1
a) We fix volume at V_1. Then heat gas at constant volume. So pressure increases to P_2. We slide weight on platform. The pressure P_2 can now hold the weight mg.
b) We continue to heat the gas until volume increases to V_2. This raises the weight a height h.



c) we fix the volume at V_2. Slide the weight over. remove the heat. Pressure will decrease at constant volume, V_2, down to P_1

a) $P_1 \to P_2$, V_1 constant
b) $V_1 \to V_2$, P_2 constant
c) $P_2 \to P_1$, V_2 constant
d) $V_2 \to V_1$, P_1 constant



0) At equilibrium, P_1, V_1, T_1

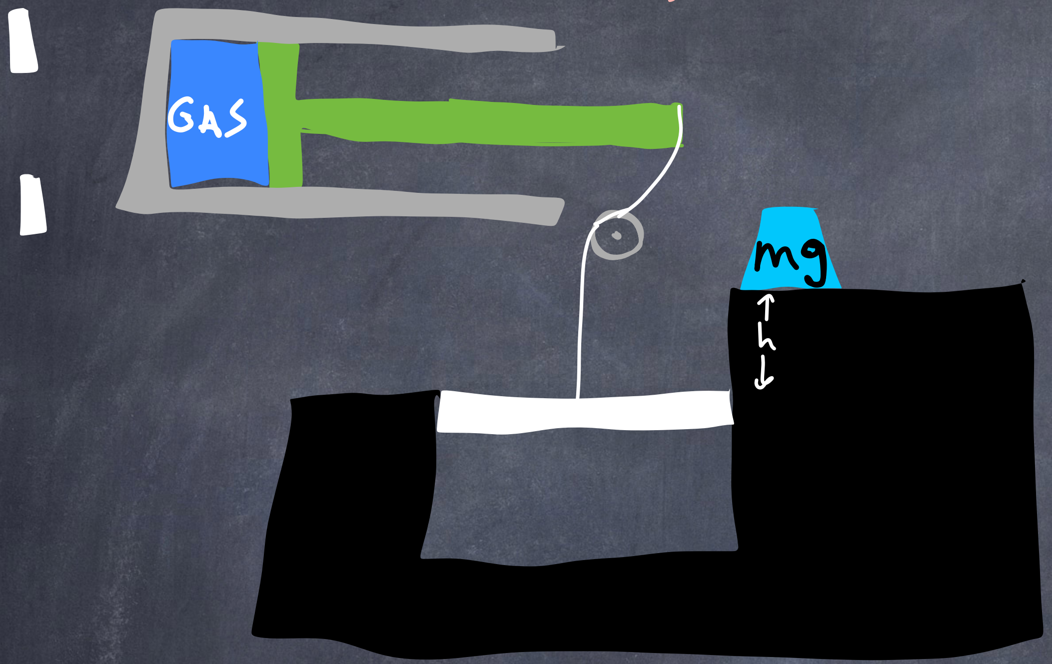
a) We fix volume at V_1. Then heat gas at constant volume. So pressure increases to P_2. We slide weight on platform. The pressure P_2 can now hold the weight mg.

b) We continue to heat the gas until volume increases to V_2. This raises the weight a height h.

c) We fix the volume at V_2. Slide over the weight. Remove the heat. Pressure will decrease at constant volume V_2 to P_1

d) Unfix the volume, continue to remove heat. The volume will decrease at constant pressure P_1, down to V_1. This lowers the platform.

a) $P_1 \to P_2$, V_1 constant
b) $V_1 \to V_2$, P_2 constant
c) $P_2 \to P_1$, V_2 constant
d) $V_2 \to V_1$, P_1 constant



initial) At equilibrium, P_1, V_1, T_1

a) We fix volume at V_1. Then heat gas at constant volume. So pressure increases to P_2. We slide weight on platform. The pressure P_2 can now hold the weight mg.

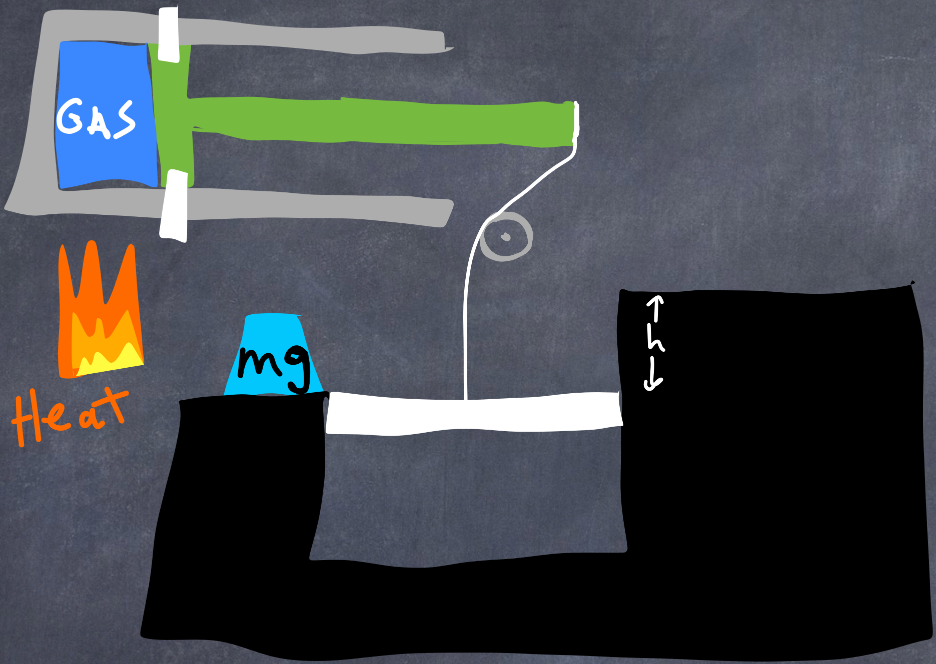
b) We continue to heat the gas until volume increases to V_2. This raises the weight a height h.

c) We fix the volume at V_2. Slide over the weight. Remove the heat. Pressure will decrease at constant volume V_2 to P_1.

d) Unfix the volume. We continue to allow heat to be removed. The volume will decrease at constant pressure P_1 to V_1.

final = initial) P_1, V_1, T_1, platform is in original position, but we've done work $W = mgh$

Summary:



cycle:

a: heat at fixed volume, pressure increases.

b: heat at fixed pressure, volume increases. work(+)

c: cool at fixed volume, pressure decreases.

d: cool at fixed pressure, volume decreases. work(−)

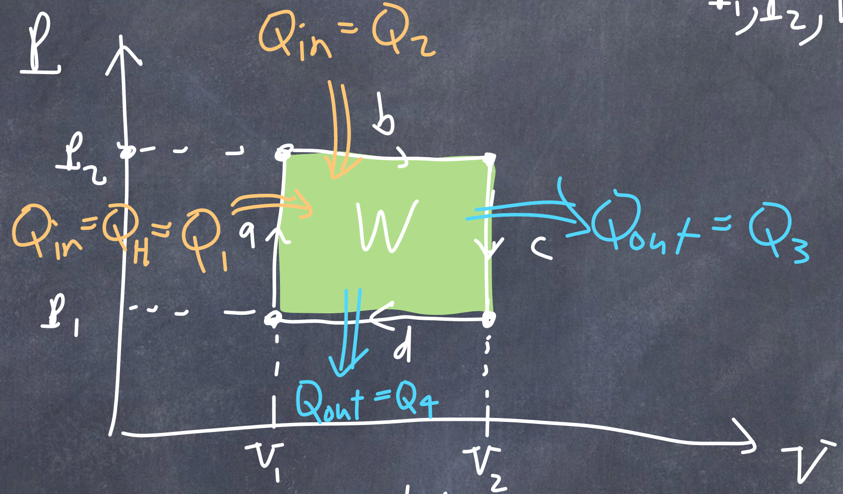
Total work done by system = $\bar{F} \cdot \bar{x} = F_g h = mgh \Rightarrow W = mgh$

Cycle:

a: heat at fixed V, P increases
b: heat at fixed P, V increases
c: cool at fixed V, P decreases
d: cool at fixed P, V decreases

Draw: P vs. V cycle, showing heat coming in and out, show the work.

Calculate: ΔU, W, Q_{in}, Q_{out}. How do $P_1, P_2, V_1, V_2, Q_{in}, Q_{out}$ relate to h.
(↑ height)



$\Delta U = 0$

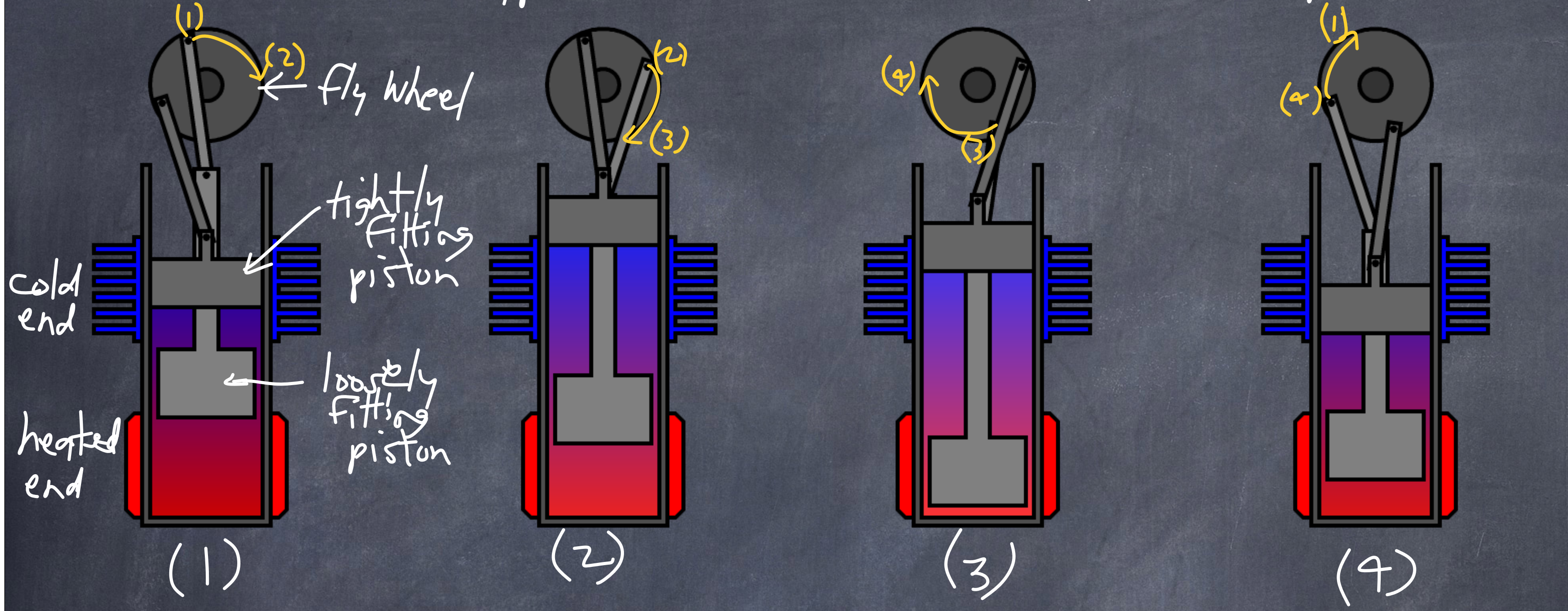
work done: $\overset{b\,(+)}{P_2(V_2 - V_1)} - P_1(V_2 - V_1) = (P_2 - P_1)(V_2 - V_1)$

$Q_{in} = Q_1 + Q_2$ heat into system (+)

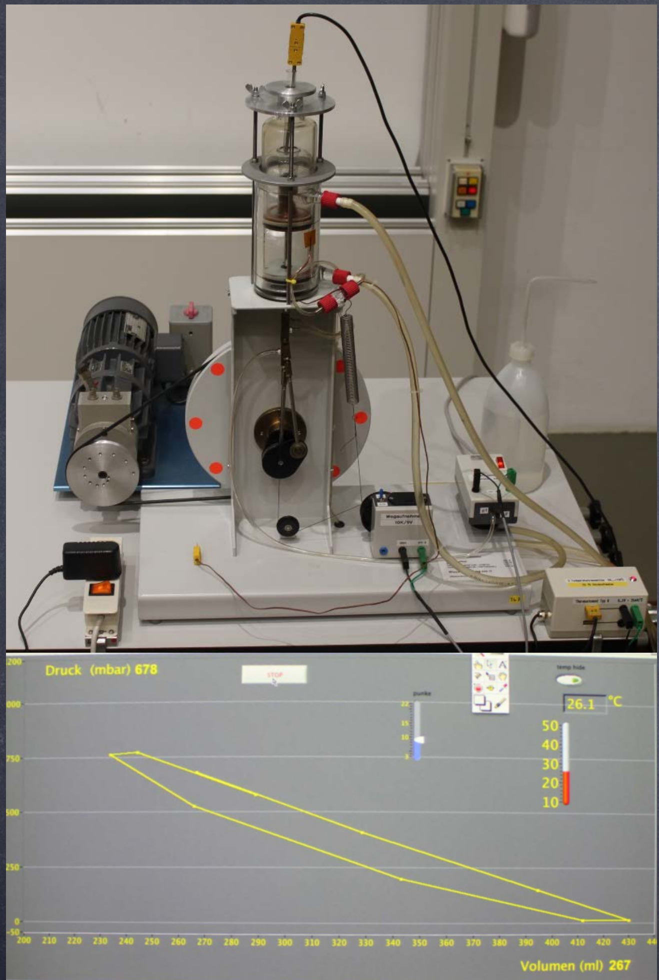
$Q_{out} = Q_3 + Q_4$ heat out system (−)

$W = mgh$ = area of PV diagram $= (P_2 - P_1)(V_2 - V_1) = Q_{in} - Q_{out}$

stirling "Beta type" motor : 1 chamber + 2 pistons.

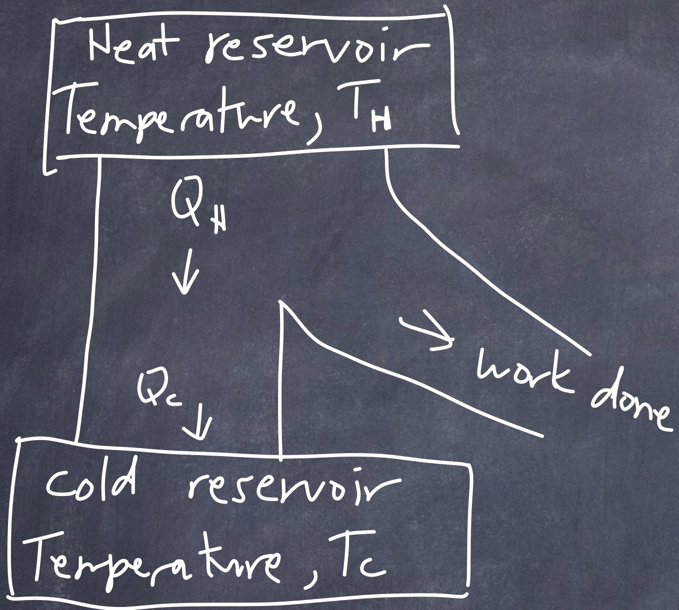


1) Most gas is in the hot end. The gas increase in pressure from heat and expands into the colder area. (2) The tight piston is pushed up (power stroke) The wheel is spinning, and pushes the loose piston down. This moves the hot gas to the cold end. The hot gas is cooled, causing gas to contract, so the tight piston is pulled down. The wheel continues to spin, pulling the loose piston up. Back to (1)



Druck (mbar) 678
26.1 °C
50
40
30
20
10
750
500
250
0
-50
200 210 220 230 240 250 260 270 280 290 300 310 320 330 340 350 360 370 380 390 400 410 420 430
Volumen (ml) 267

Heat engine (heat converted to work)



In a cycle, initial & final state are the same, so no change in internal energy (no change in U)

from 1st law of thermodynamics,

$$Q = \cancel{\Delta U} + W$$

$$W = Q_H - |Q_c|$$

(avoids confusion with | |)

The efficiency is defined as the work divided by the heat taken from the hot reservoir:

$$\epsilon = \frac{W}{Q_H} = \frac{Q_H - |Q_c|}{Q_H} = 1 - \frac{|Q_c|}{Q_H}$$

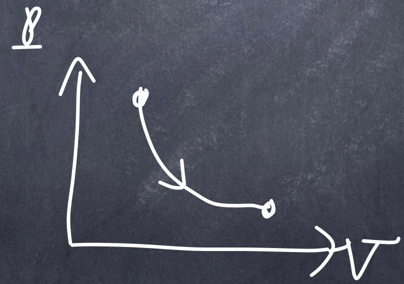
This is the maximum possible efficiency

The 2nd law of thermodynamics for heat engines:
It is impossible for a heat engine to convert 100% of heat from a heat source (at constant temp.) into work energy. * caveat

Typical efficiencies:

steam engine ~ 40%
internal combustion engine ~ 25%
Formula One engine ~ 47%
rocket engine ~ 70%

* It is possible during a thermal expansion step, but <u>not</u> in a cycle.



What is the maximum possible efficiency of a heat engine cycle? We can calculate this for a reversible process, (no energy lost to friction, no heat conduction, no radiation)

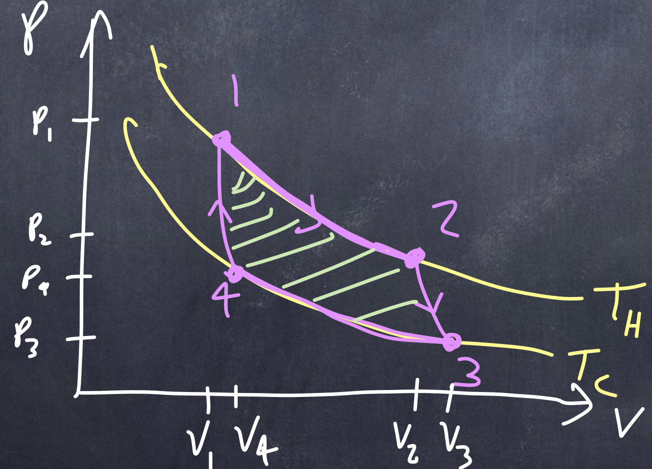
All reversible engines have the same efficiency. So we just need one case to calculate. Solved by Carnot in 1824; he used an ideal gas cycle.

1: P_1, V_1, T_H

2: P_2, V_2, T_H

3: P_3, V_3, T_C

4: P_4, V_4, T_C



$1 \to 2$: isothermal expansion $\Delta U = 0$

$2 \to 3$: adiabatic expansion $Q = 0$

$3 \to 4$: isothermal compression $\Delta U = 0$

$4 \to 1$: adiabatic compression $Q = 0$

efficiency of cycle $\epsilon = 1 - \frac{|Q_c|}{Q_H}$

$1 \to 2$: $Q_H = W = \int_{V_1}^{V_2} P\,dV = nRT_H \int_{V_1}^{V_2} \frac{dV}{V} = nRT_H \ln \frac{V_2}{V_1}$
$(\Delta U = 0)$

$3 \to 4$: $|Q_c| = |-W| = nRT_c \ln \frac{V_3}{V_4}$

Look at $2 \to 3$ + $4 \to 1$: these are adiabatic processes:

$TV^{\gamma-1} = \text{constant}$ $\quad PV^{\gamma} = \text{constant}$

$2 \to 3$: $T_H V_2^{\gamma-1} = T_c V_3^{\gamma-1}$ $\quad \frac{T_H}{T_c} = \frac{V_3^{\gamma-1}}{V_2^{\gamma-1}}$

$4 \to 1$: $T_c V_4^{\gamma-1} = T_H V_1^{\gamma-1}$ $\quad \frac{T_H}{T_c} = \frac{V_4^{\gamma-1}}{V_1^{\gamma-1}}$

It must be that $\frac{T_H}{T_c} = \frac{V_3}{V_2} = \frac{V_4}{V_1} \Rightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1}$

$$\epsilon = 1 - \frac{|Q_c|}{Q_H} = 1 - \frac{\cancel{nR}\, T_c \cancel{\ln \frac{V_3}{V_4}}}{\cancel{nR}\, T_H \cancel{\ln \frac{V_2}{V_1}}}$$

the same

$$\epsilon = 1 - \frac{T_c}{T_H}$$

ϵ_c: The Carnot efficiency:

$$\epsilon_c = 1 - \frac{T_c}{T_H} \quad \text{where} \quad \frac{T_c}{T_H} = \frac{|Q_c|}{Q_H}$$

This is the efficiency for a <u>perfect</u> reversible engine.

The Carnot efficiency cannot be beat! Any efficiency higher violates the 2nd law of thermodynamics.

The maximum efficiency only depends on the temperature difference.

$$\epsilon_{SL} = \frac{\text{actual efficiency}}{\text{Carnot efficiency}} = \frac{\epsilon}{\epsilon_c}$$

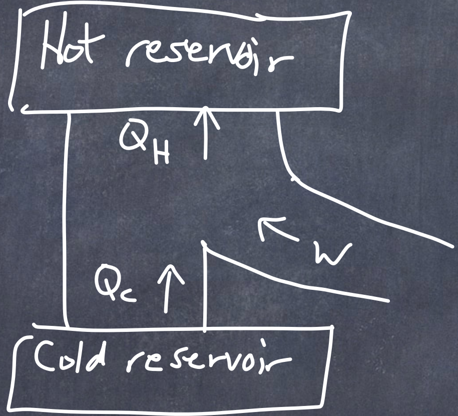
↑ efficiency with respect to the best efficiency (from the 2nd Law)

Example: An engine has 600 K high temperature, and a 300 K low temperature, and its 30% efficient. What is ϵ_c + ϵ_{SL}?

It's $\epsilon_c = 1 - \frac{300\,K}{600\,K} = \frac{1}{2}$ $\quad \epsilon_{SL} = \frac{\epsilon}{\epsilon_c} = \frac{30\%}{1/2} = 60\%$

A refrigerator is like a heat engine, but running backwards. Work is done (in to system) to extract heat from a cold reservoir. (The piston is used to lower the pressure in one cylinder → lowers temperature)

$$|Q_H| = |Q_c| + W$$



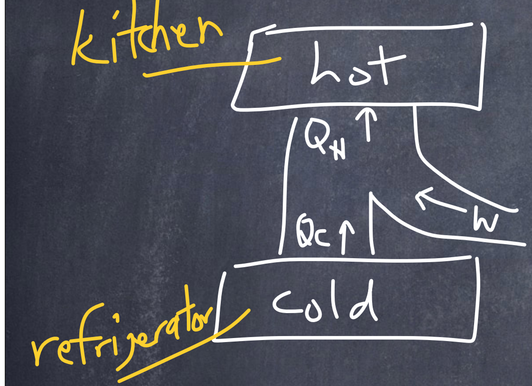
2nd law of thermodynamics for a fridge. It is impossible for a refrigerator cycle to only transfer heat from a cold object to a hot object (work needs to be done to do this

The measure of performance of a refrigerator is

C.O.P. = coefficient of performance $= \dfrac{Q_c}{W}$

2nd law: C.O.P. must <u>not</u> be ∞

For a typical refrigerator, C.O.P. ~ 5.5.
How much work + power is needed to make ice cubes from 1 liter of water at 10°C?
How much heat do we put into our kitchen doing this?



How much heat do we need to remove to make the ice?

$$Q_c = Q_1 + Q_2 = 375 \text{ kJ}$$

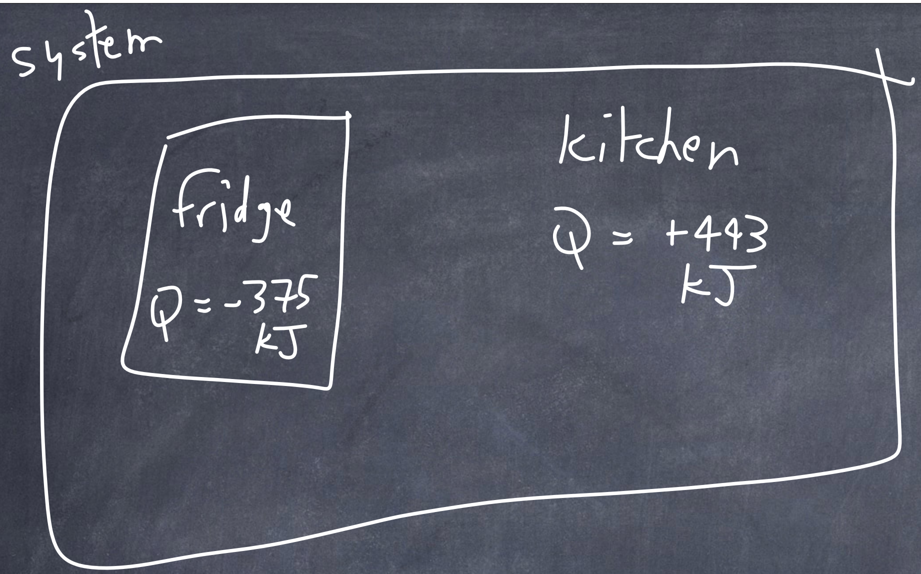
$Q_1 = mc\Delta T$, $Q_2 = mL_f$

$$W = \frac{Q_c}{\text{C.O.P.}} = \frac{375 \text{ kJ}}{5.5} = 68 \text{ kJ}$$

$$Q_H = \text{heat into kitchen} = |Q_c| + W = 375 \text{ kJ} + 68 \text{ kJ} = 443 \text{ kJ}$$

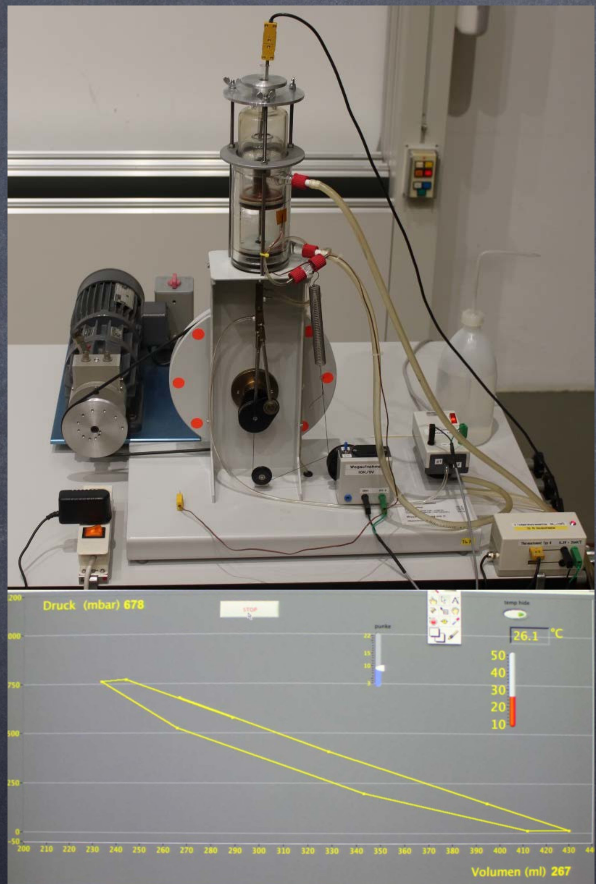
$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{68 \text{ kJ}}{10 \text{ seconds}} = 6.8 \text{ kW}$$

typically, a refrigerator uses ~ 25 watts



$$Q_{total} = Q_{fridge} + Q_{kitchen}$$
$$= -375 + 443 \text{ kJ}$$
$$= +68 \text{ kJ}$$

⇒ You can't cool a room by opening a fridge.



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STOP
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temp hide
26.1 °C
50
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750
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250
0
-50
200 210 220 230 240 250 260 270 280 290 300 310 320 330 340 350 360 370 380 390 400 410 420 430
Volumen (ml) 267