Sensitivity of Pulsar Timing Arrays towards Polarizations of Gravitational Waves

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Abstract

We extend our investigation[1] on the sensitivity towards polarizations of gravitational waves of current and future experiments (LIGO, LISA, ET, DECIGO) to pulsar timing arrays (PTA). The focus of our upcoming paper lies on giving a complete derivation[2, 3] of the redshift formula for all six possible polarizations including modifications to general relativity[4].

Theory

The linearization of the Einstein field equations leads to a linear wave equation for perturbations in the metric. Considering the symmetries of the metric and the field equations, there can be at most 6 polarizations. The vacuum equation describing the free propagation of gravitational waves (GW) in General Relativity (GR) has an additional symmetry, which further reduces the degrees of freedom to the two tensor polarizations.

The GR action is given by:

$$S[g] = \frac{1}{16\pi G}\int R\sqrt{-g}d^4x. \quad (1)$$

One can include additional scalar and vector fields which do not interact with the matter fields of the theory. This will effectively act as a modification of the theory of gravity. The presence of such a scalar or vector field allows for additional gravitational wave polarizations as depicted in Fig. 1, since they break the vacuum symmetry.

The Lagrangian for GR modified with a scalar field is given by [4]:

$$S[g,\phi] = \frac{1}{16\pi G}\int \left[R - 2g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right]\sqrt{-g}d^4x. \quad (2)$$

The modification with a vector field yields the following Lagrangian:

$$S[g,V] = \frac{1}{16\pi G}\int \Big[(1+\omega V_\mu V^\mu)R - K^{\mu\nu}_{\rho\sigma}\nabla_\mu V^\rho \nabla_\nu V\sigma + \lambda(V_\mu V^\mu + 1)\Big]\sqrt{-g}d^4x. \quad (3)$$

The metric perturbation due to a gravitational wave in its most general form, described in terms of the tensor $(+,\times)$, vector (x,y) and scalar (b,l) polarizations takes the following form:

$$h_{ij}(t,\vec{x}) = \begin{pmatrix} h_b + h_+ & h_\times & h_x \\ h_\times & h_b - h_+ & h_y \\ h_x & h_y & h_l \end{pmatrix} e^{2\pi i f\left(t - \frac{\hat{\Omega}\cdot\vec{x}}{c}\right)} + c.c. \quad (4)$$

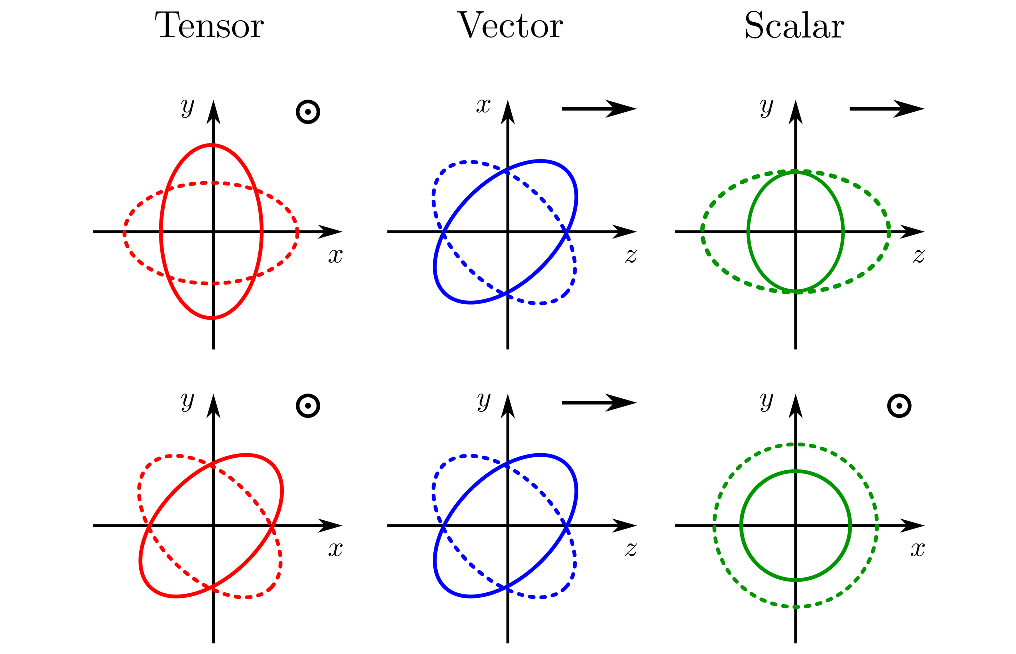
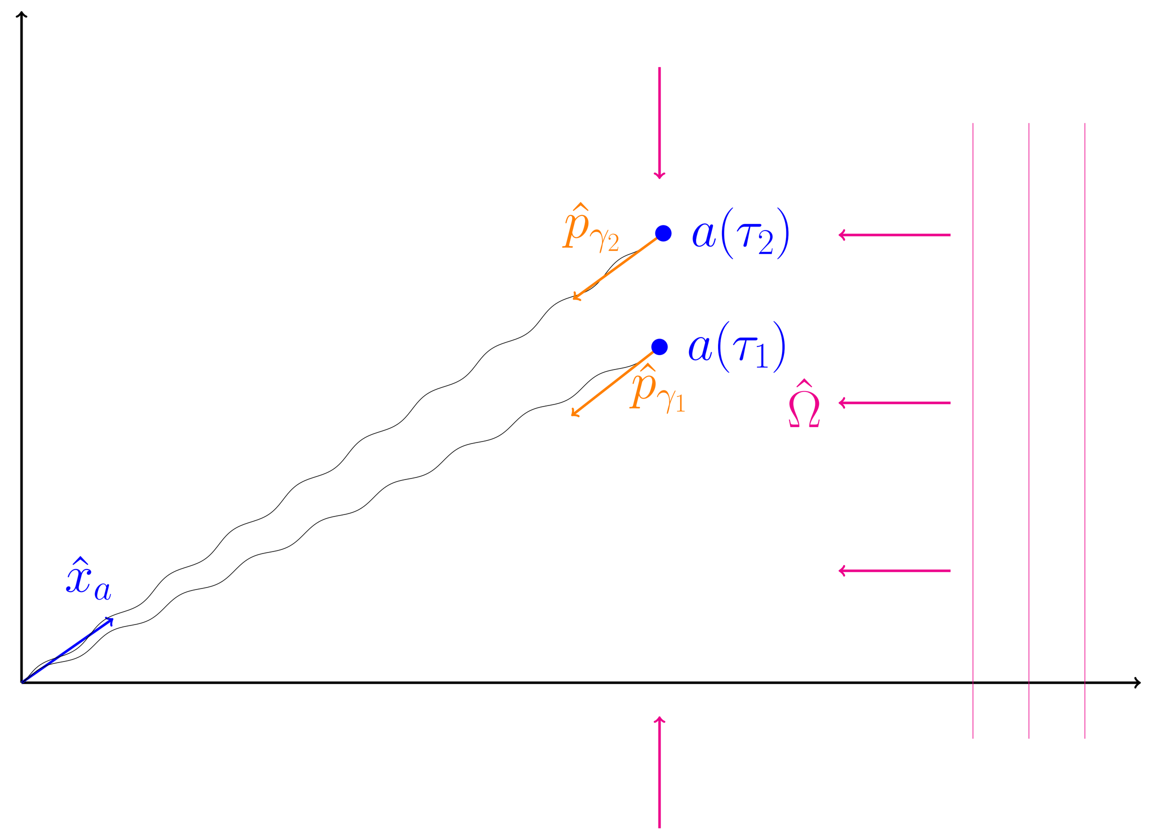


Figure 1: All possible polarizations of gravitational waves.

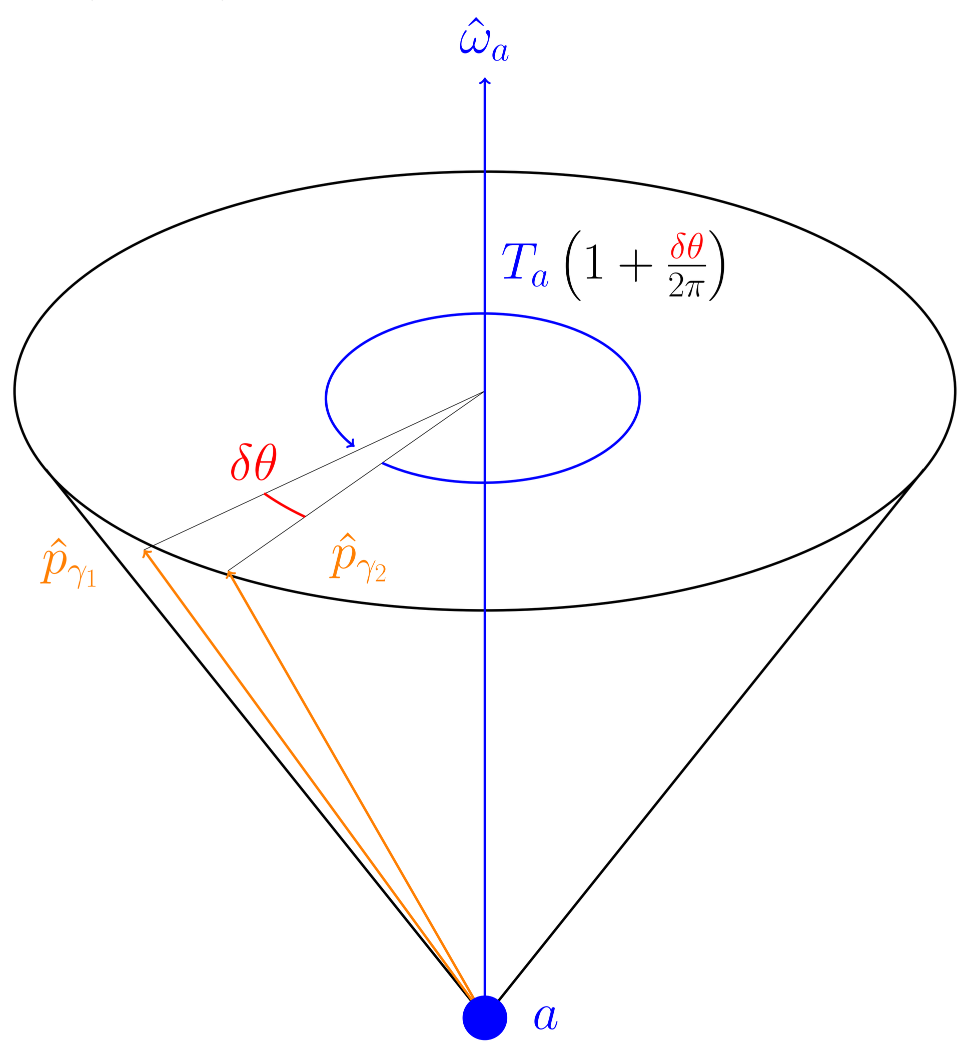
Photon Geodesics

When a gravitational wave passes through our galaxy it changes the distance between Earth and a pulsar a. This causes a redshift in in the frequency of the pulses arriving at Earth. Pulsar timing array (PTA) experiments measure the redshift in the pulses of a collection of pulsars to detect a gravitational waves.

To derive the redshift of these pulses we need to track the photons of the pulsar jets of two subsequent pulses (when the jet points at Earth) from the pulsar to Earth and compare their arrival times.



At zeroth order the arrival time difference is given by the pulsar period T_a. In the presence of a gravitational wave however, two subsequent photons cover different distances due to different emission angles. Thus the time between the two photon emissions is given by $T_a\left(1 + \frac{\delta\theta}{2\pi}\right)$.



$$z_P = \frac{\Delta T}{T_a} = \frac{t_{\mathcal{R},\gamma_2} - t_{\mathcal{R},\gamma_1} - T_a}{T_a} \overset{\mathcal{O}(\omega T_a)}{\approx} \sum_A F^A \Delta h_A(t_0) - \frac{\omega T_a}{4}\sum_A \frac{\Delta h^s_A(\lambda_a)}{1+\gamma} + \frac{\delta\theta}{2\pi}, \quad (5)$$

with

$$\Delta h_A = \frac{h_A}{2}\frac{\cos(\varphi_0 + \varphi_A) - \cos\left(\varphi_0 + \varphi_A - \frac{L\omega}{c}[1+\gamma]\right)}{[1+\gamma]} \quad (6)$$

Overlap reduction functions

The overlap reduction functions describe the geometric part of the two point correlation of signals from two pulsars. In the literature the exponential terms are dropped, with the reasoning that their contribution is small for large $\frac{L\omega}{c}$. It is of order 10^3 and can thus be considered as large, however the exponential terms are canceling the singularities due to the denominators.

We calculate the full integral to see whether it indeed converges to the Hellings & Downs curve for large $\frac{L\omega}{c}$. The evaluation of the exact expression becomes significantly more calculation intensive for larger $\frac{L\omega}{c}$. Up to now our resulst are inconclusive.

$$\Gamma_T = \frac{3\pi}{4}\sum_A \int_{\mathbb{S}^2} F^A_a(\hat{\Omega})F^A_b(\hat{\Omega})\frac{1 - e^{i\omega\tau_a}}{1+\gamma_a}\frac{1 - e^{-i\omega\tau_b}}{1+\gamma_b}d\hat{\Omega} \quad (7)$$

with

$$\tau_a = \frac{L}{c}(1+\gamma_a), \qquad \gamma_a = \hat{\Omega}\cdot\hat{x}_a. \quad (8)$$

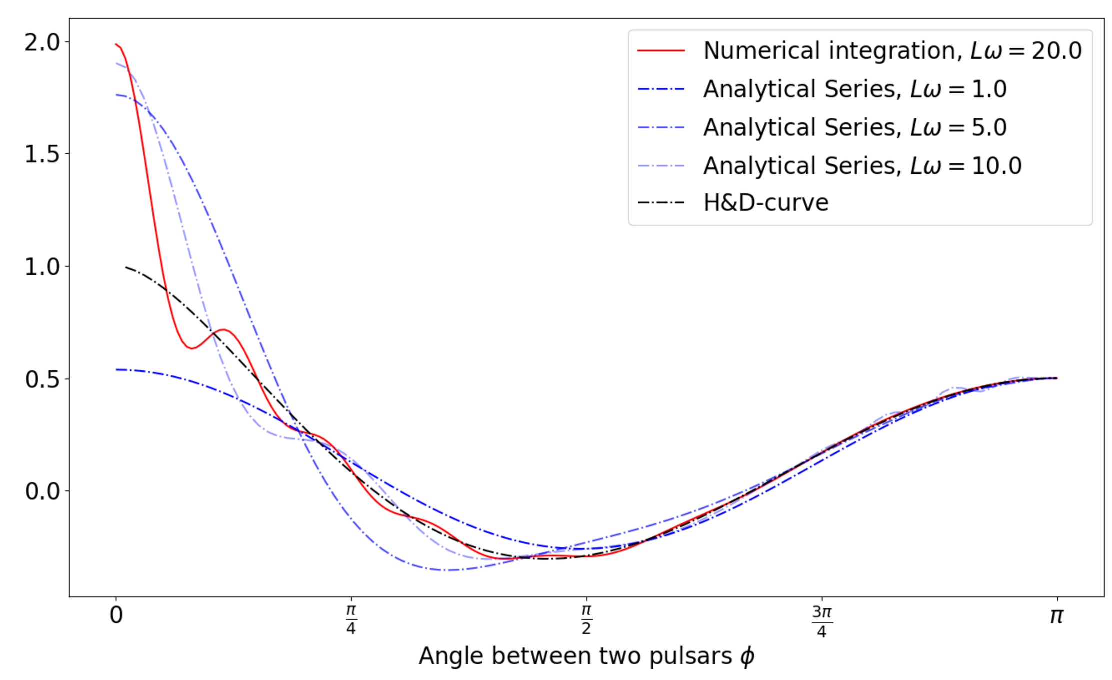


Figure 2: Comparison of the exact solution which is given by a series at different $\frac{L\omega}{c}$ with a numerical integration and the Hellings & Downs curve.

Conclusions

- The PTA's are using milisecond pulsars $T_a \approx 10^{-3}$ s and they measure in the frequency band $[10^{-9}\,\text{Hz}, 10^{-7}\,\text{Hz}]$. Thus $\omega T_a \approx 10^{-11}$, while the gravitational wave background is expected to be of order $h \approx 10^{-15}$. This means that the correction term proportional to $\omega T_a h$, which includes $\delta\theta$ is between first and second order.
- For most of the angles ϕ the exact expression for the overlap reduction function seems to converge to the Hellings & Downs curve but it is still unclear what happens around $\phi = 0$.

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