Logarithmic accuracy of parton showers

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`1805.09327`
`in collaboration with`
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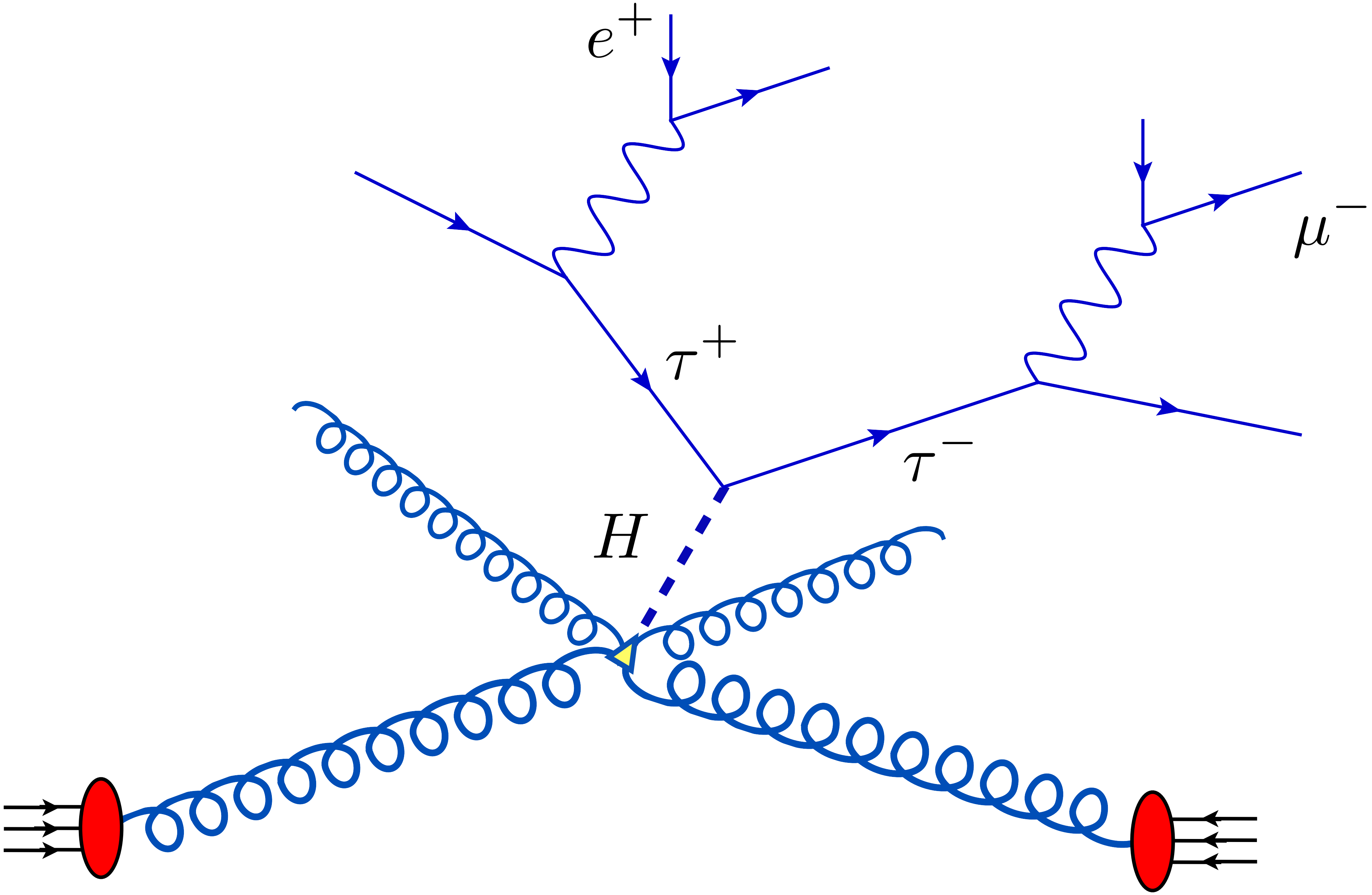
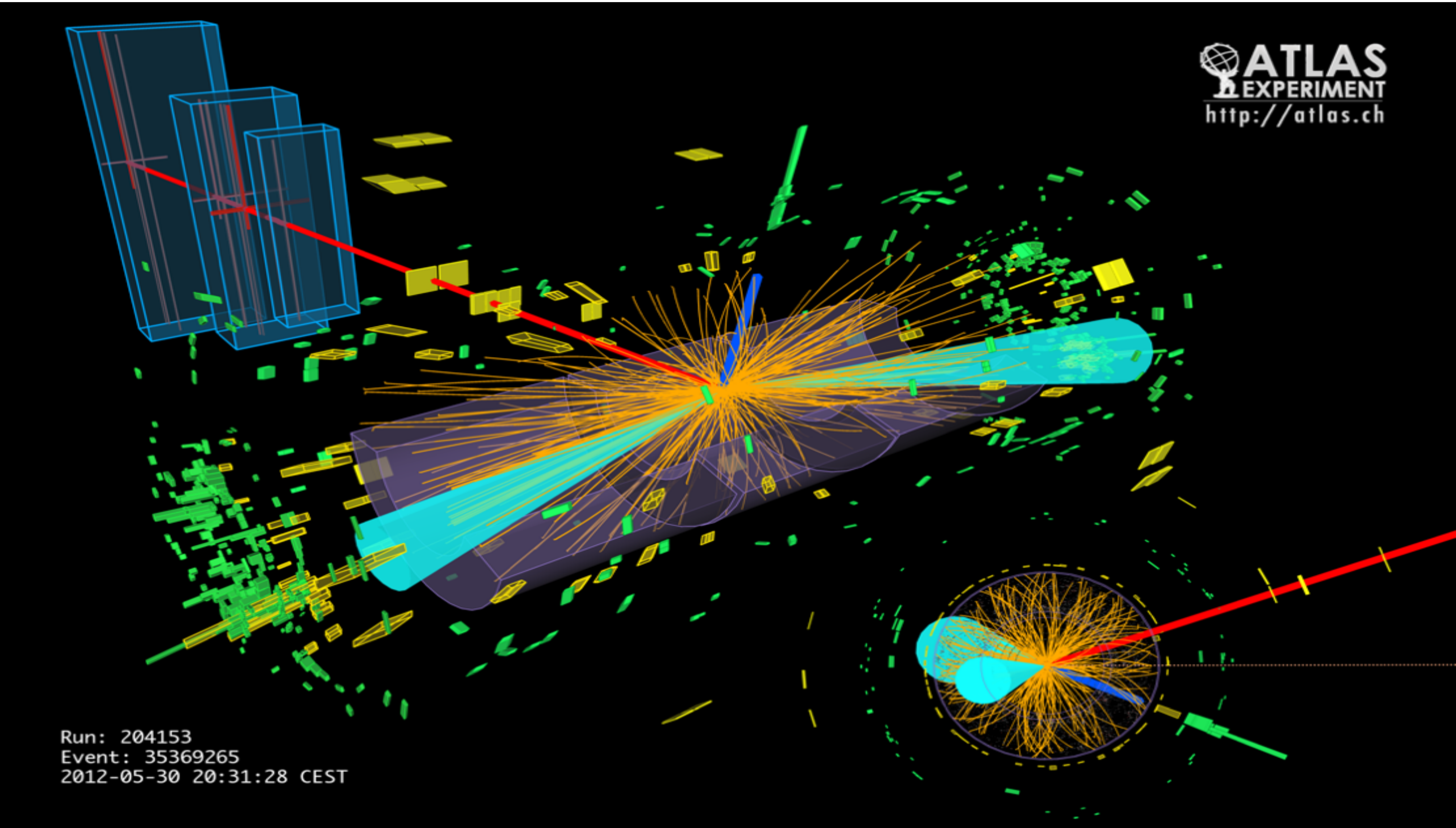
- Parton Shower algorithms simulate the evolution of QCD systems from the hard scattering down to the energies of the hadrons observed in the detector
  - i.e. they get from this picture…

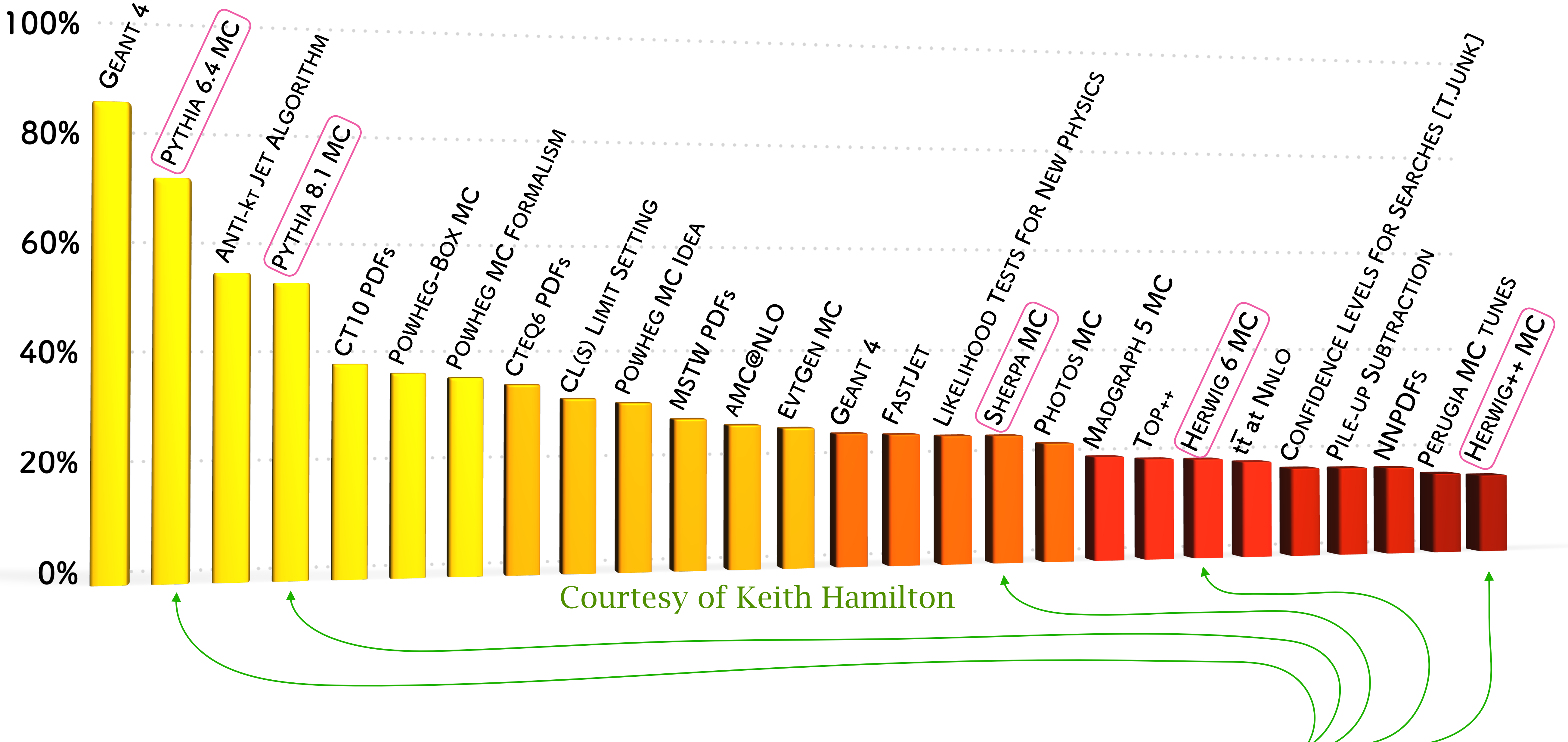


- Parton Shower algorithms simulate the evolution of QCD systems from the hard scattering down to the energies of the hadrons observed in the detector

  - …to this one



- Percentage of ATLAS+CMS+LHCb papers citing a given article since Jan ’14 (w/o self citations)



Courtesy of Keith Hamilton

- Parton Showers are central to the LHC programme: realistic event simulations

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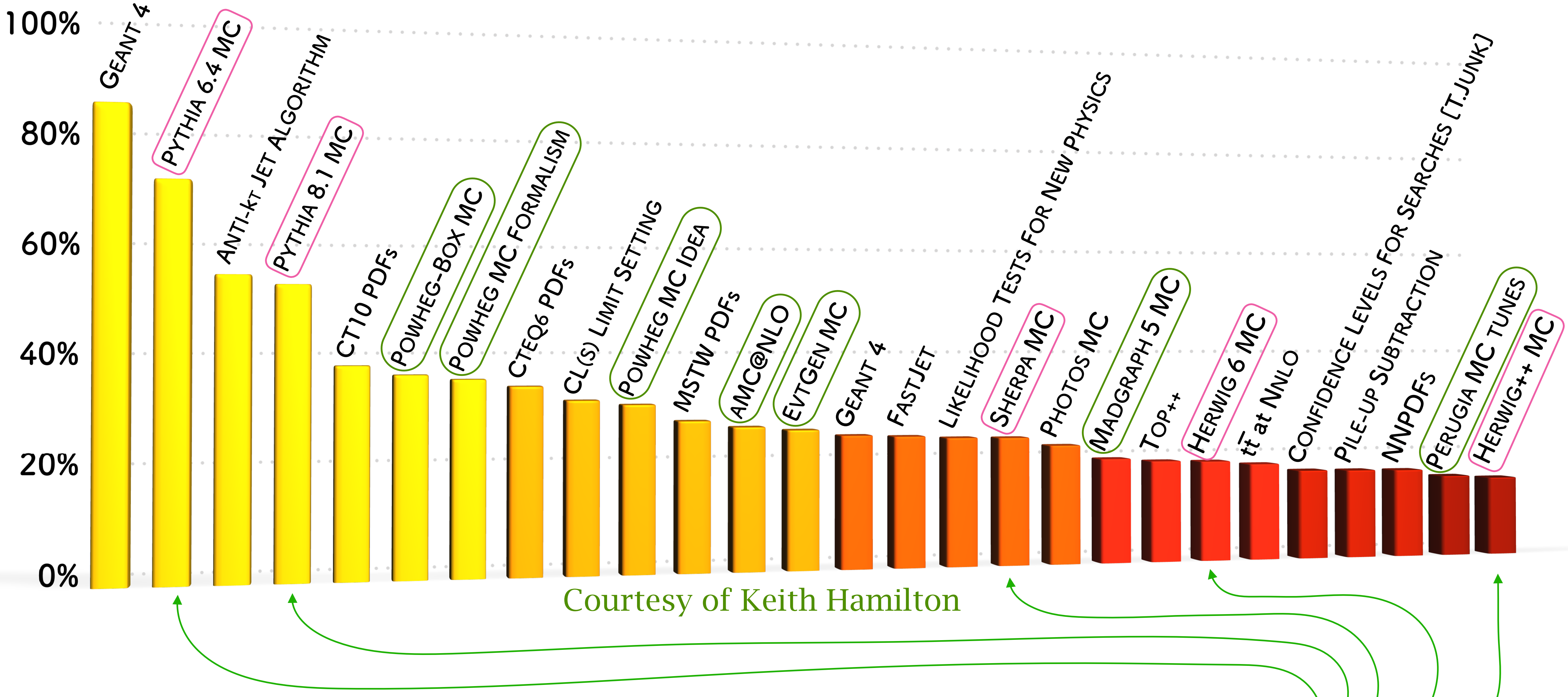

Courtesy of Keith Hamilton

- Parton Showers are central to the LHC programme: realistic event simulations

- Used in essentially all event generators

An example: the top pole mass

- Template method in ttbar —> 2 lepton + 4 jets

$$m_{\text{top}} = 172.08 \pm 0.39\,(\text{stat}) \pm 0.82\,(\text{syst})\ \text{GeV}$$

[ATLAS 1810.01772]

- **Shower uncertainty constitutes a substantial fraction of the TH error**

[Ravasio, Jezo, Oleari, Nason '18]

change matching: POWHEG <—> MC@NLO

change shower: Pythia <—> Herwig

vary generator and shower parameters

	$\sqrt{s}$ = 7 TeV	$\sqrt{s}$ = 8 TeV	
Event selection	Standard	Standard	BDT
m_{top} result [GeV]	172.33	171.90	172.08
Statistics	0.75	0.38	0.39
– *Stat. comp.* (m_{top})	0.23	0.12	0.11
– *Stat. comp.* (JSF)	0.25	0.11	0.11
– *Stat. comp.* (bJSF)	0.67	0.34	0.35
Method	0.11 ± 0.10	0.04 ± 0.11	0.13 ± 0.11
Signal Monte Carlo generator	0.22 ± 0.21	0.50 ± 0.17	0.16 ± 0.17
Hadronization	0.18 ± 0.12	0.05 ± 0.10	0.15 ± 0.10
Initial- and final-state QCD radiation	0.32 ± 0.06	0.28 ± 0.11	0.08 ± 0.11
Underlying event	0.15 ± 0.07	0.08 ± 0.15	0.08 ± 0.15
Colour reconnection	0.11 ± 0.07	0.37 ± 0.15	0.19 ± 0.15
Parton distribution function	0.25 ± 0.00	0.08 ± 0.00	0.09 ± 0.00
Background normalization	0.10 ± 0.00	0.04 ± 0.00	0.08 ± 0.00
W+jets shape	0.29 ± 0.00	0.05 ± 0.00	0.11 ± 0.00
Fake leptons shape	0.05 ± 0.00	0	0
Jet energy scale	0.58 ± 0.11	0.63 ± 0.02	0.54 ± 0.02
Relative *b*-to-light-jet energy scale	0.06 ± 0.03	0.05 ± 0.01	0.03 ± 0.01
Jet energy resolution	0.22 ± 0.11	0.23 ± 0.03	0.20 ± 0.04
Jet reconstruction efficiency	0.12 ± 0.00	0.04 ± 0.01	0.02 ± 0.01
Jet vertex fraction	0.01 ± 0.00	0.13 ± 0.01	0.09 ± 0.01
b-tagging	0.50 ± 0.00	0.37 ± 0.00	0.38 ± 0.00
Leptons	0.04 ± 0.00	0.16 ± 0.01	0.16 ± 0.01
Missing transverse momentum	0.15 ± 0.04	0.08 ± 0.01	0.05 ± 0.01
Pile-up	0.02 ± 0.01	0.14 ± 0.01	0.15 ± 0.01
Total systematic uncertainty	1.04 ± 0.08	1.07 ± 0.10	0.82 ± 0.06
Total	1.28 ± 0.08	1.13 ± 0.10	0.91 ± 0.06

- Other measurements (e.g. M_W) show similar features [ATLAS 1701.07240]

- Perturbative accuracy defined in terms of how many towers of logarithms one sums up
  - e.g.

LL ~ 100% uncertainty

NLL ~ 20% uncertainty

NNLL ~ 5% uncertainty

...

Resummations

Numerical approaches (CAESAR, ARES, RadISH,...)

SCET

Factorisation theorems

Branching algorithm

Parton Showers

Pythia

Herwig

Deductor

Sherpa

DiRe

$$\Sigma(v) = \int_0^v \frac{1}{\sigma_{\mathrm{Born}}}\frac{d\sigma}{dv'}dv' \sim e^{\overset{\text{LL}}{\alpha_s^n L^{n+1}} + \overset{\text{NLL}}{\alpha_s^n L^n} + \overset{\text{NNLL}}{\alpha_s^n L^{n-1}} + \dots}$$

Resummations vs. Parton Showers

- Both frameworks provide an all-order calculation for collider observables
- Several differences in the way this is formulated
- The higher logarithmic accuracy of current resummations comes with a lower versatility

	RESUMMATIONS	PARTON SHOWERS

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Treatment of radiation	• Several simplifications: amplitudes, phase space, observable • All calculations derived in the on-shell/ singular limit (only logarithms)	• Radiation is described fully exclusively. Provide full set of final-state momenta • Full momentum conservation necessary (e.g. initial condition for hadronisation)

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Observable dependence	• Tailored to the observable, e.g. global vs. non-global, specific approximations in each case	• A simple shower should be accurate for a broad family of observables at once
Logarithmic Accuracy	• Higher logarithmic orders achieved thanks to the above simplifications in the formulation	• Currently unknown

Resummation: NLL

- To understand (and ultimately improve) the logarithmic accuracy of PS, crucial to build a systematic connection to resummation
- Use the technology of numerical resummations to approach the problem

`e.g. e+e- -> q qbar + X at NLL`

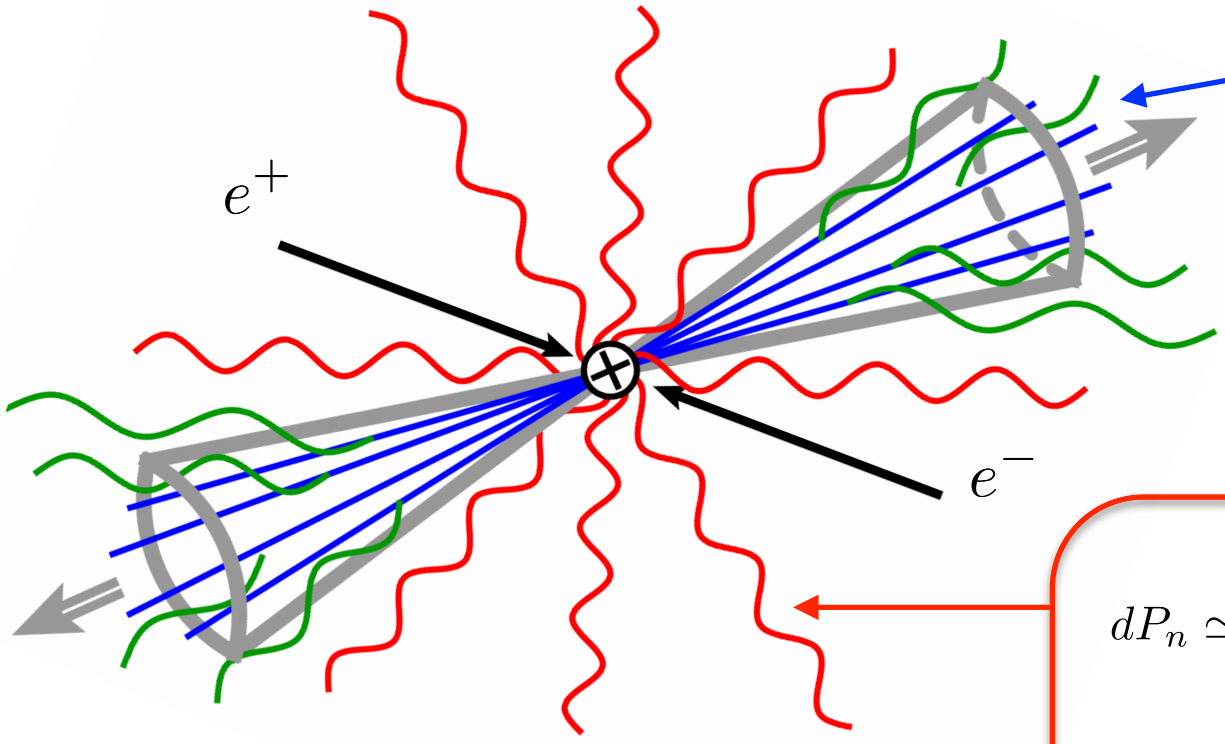

Image by T. Becher et al.

$$dP_n \simeq \frac{C_F^n}{n!}\prod_{i=1}^{n}\left(\frac{\alpha_s^{\rm CMW}(p_{\perp,i}^2)}{\pi}\frac{dp_{\perp,i}}{p_{\perp,i}}dz_i P_{q\to qg}(z_i)\frac{d\phi_i}{2\pi}\right)$$

- collinear limit described by independent emissions strongly separated in angle

[Catani et al. ’91-’93; Banfi, Salam, Zanderighi ’01-’04]

$$dP_n \simeq \frac{1}{n!}\prod_{i=1}^{n}\frac{\alpha_s}{\pi}\frac{d\omega_i}{\omega_i}\frac{d^2\Omega}{4\pi}N_c\sum_{\pi_n}\frac{p_1\cdot p_2}{(p_1\cdot k_{i_1})(k_{i_1}\cdot k_{i_2})\dots(k_{i_n}\cdot p_2)}$$

- soft wide angle limit described by a collection of *soft* colour dipoles strongly ordered in energy (planar limit)

[Dasgupta, Salam ’01; Banfi, Marchesini, Smye ’02]

Parton Showers

- Main defining features (at least for LO showers)

1. Ordering variable: generate emissions in sequence according to a kinematic variable v (e.g. k_t, angle, virtuality).

2. Branching probability: state S_n with n partons at a given v found with a probability $P(S_n, v)$

   ➡ This probability evolves with the ordering variable as

$$\frac{dP(S_n, v)}{d\ln 1/v} = -f(S_n, v)P(S_n, v)$$

This evolution equation accounts for real and virtual corrections (unitarity)

3. Kinematic mapping: state S_{n+1} obtained from a state S_n via a mapping $\mathcal{M}(S_n \to S_{n+1}; v)$

   ➡ Is a function of all partons involved in the branching. It defines how the recoil is absorbed by other partons in the event. E.g. for a *local* recoil scheme

$$S_{n+1} = \mathcal{M}(S_n, v; \underbrace{i, j}_{\text{emitters}}, \underbrace{z, \phi}_{\text{emission}})$$

   ➡ The map is accompanied by the relative probabilities of all possible new states, i.e.

$$f(S_n, v) = \sum_{i,j} \int dv' dz d\phi \frac{d\mathcal{P}(S_n, v'; i, j, z, \phi)}{dv' dz d\phi} \delta(\ln v'/v) \qquad \sum_{i,j} d\mathcal{P}(S_n, v; i, j, z, \phi) \simeq \frac{d\Phi_{n+1}}{d\Phi_n} \frac{|M^2(S_{n+1})|}{|M^2(S_n)|}$$

A case study: dipole showers

- Several designs available…

global recoil

local recoil

angular ordering

kt ordering

virtuality ordering

antenna shower

dipole shower

A case study: dipole showers

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- We focus on k_t-ordered dipole showers with local recoil
  - Most common design today
  - Ability to reproduce non-global logarithms at LL, for which different solutions might fail

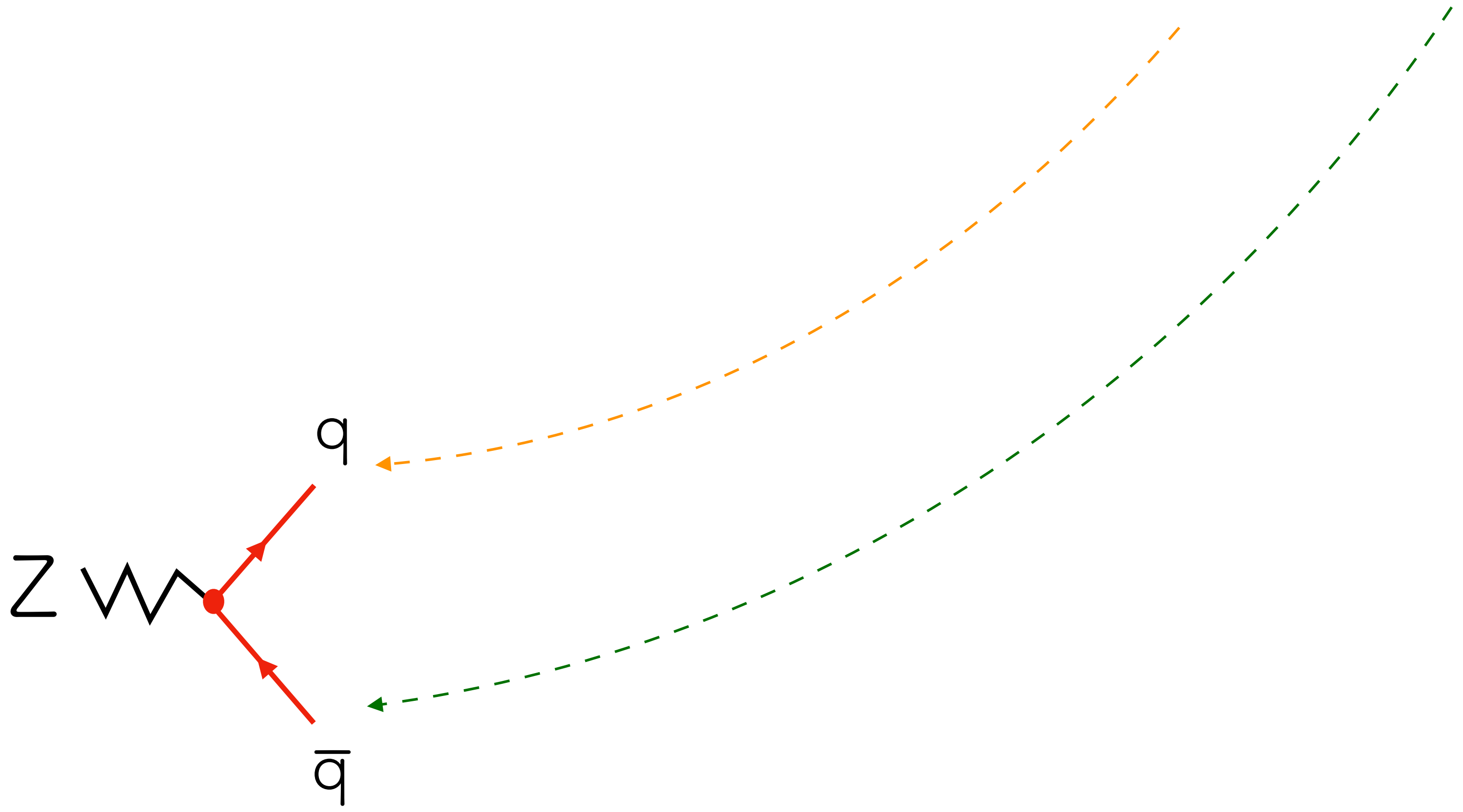
see e.g. [Banfi, Corcella, Dasgupta ’06]

- Consider the designs of `Pythia8`’s shower and `Dire` as a case study

[Sjostrand, Skands ’04]
[Hoeche, Prestel ‘15]

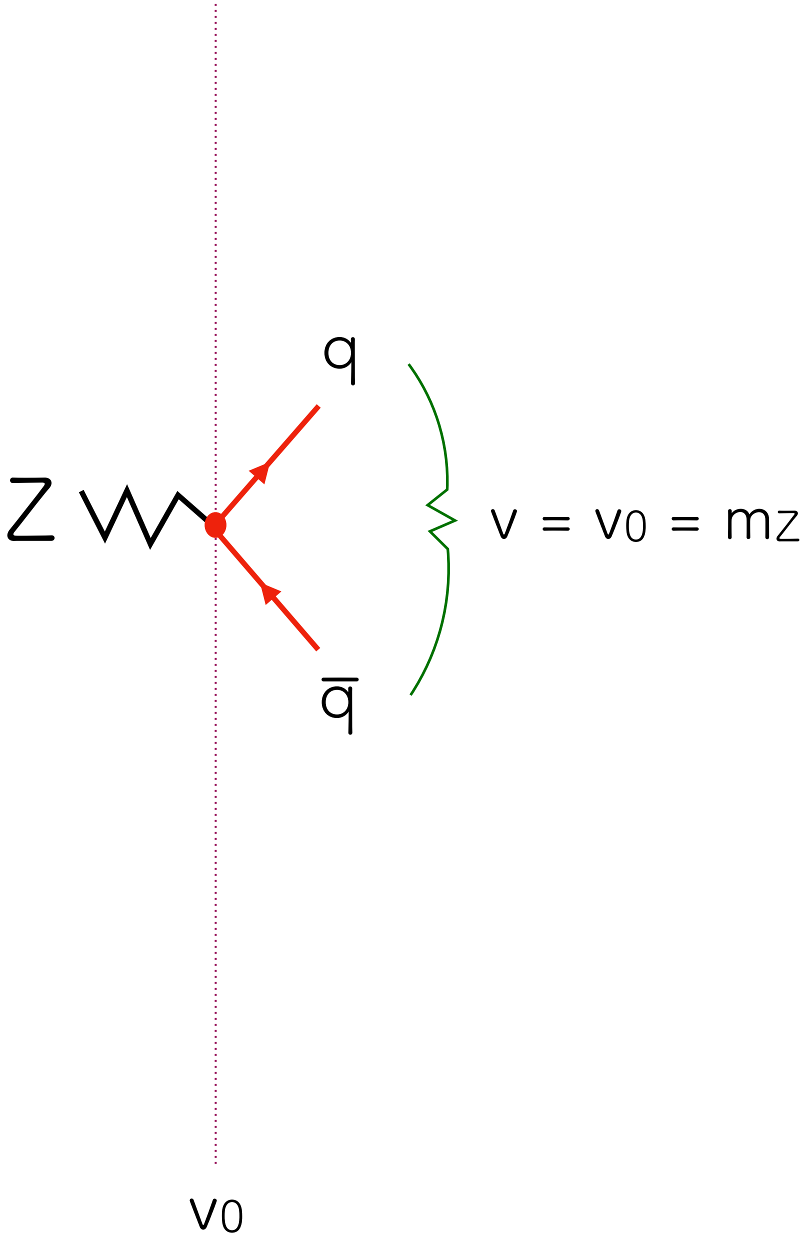
Dipole showers

- Events are viewed throughout as a collection of colour-anticolour dipole ends



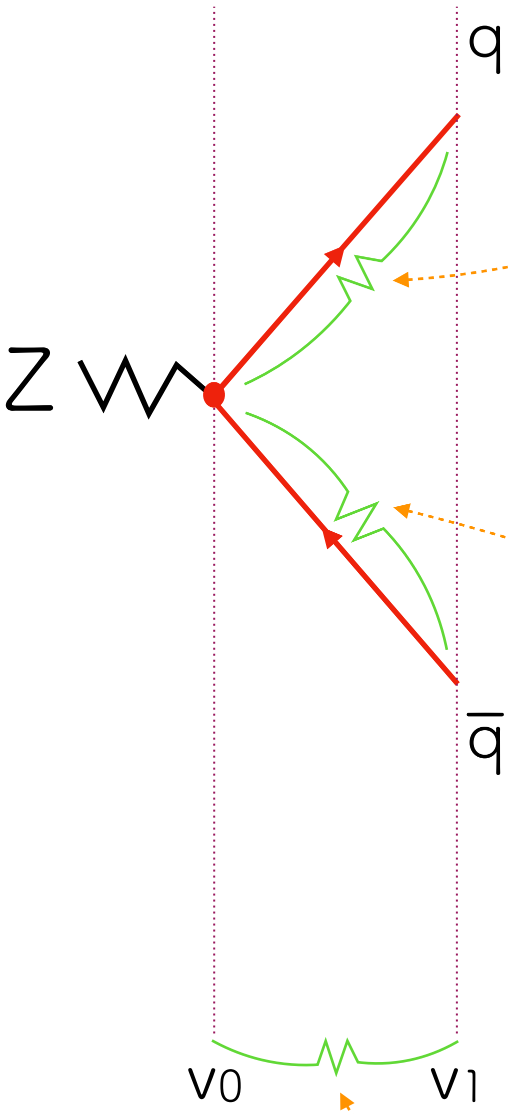
Dipole showers: evolution variable

- Ordering variable v : smallest $p_\perp$ separation (resolution) between any pair of partons
- Zooming out to smaller v values more partons get resolved



Dipole showers: branching

- Branching probability: evolution equation solved in terms of a Sudakov form factor



P[q dipole end doesn't emit in $v_0 \rightarrow v_1$]

$$\Delta_q\left(v_0, v_1\right) = \exp\left[-\int_{v_1}^{v_0} d\mathcal{P}_{q[\bar{q}]\to qg[\bar{q}]}\right]$$

P[q dipole end doesn't emit in $v_0 \rightarrow v_1$]

$$\Delta_{\bar{q}}\left(v_0, v_1\right) = \exp\left[-\int_{v_1}^{v_0} d\mathcal{P}_{\bar{q}[q]\to \bar{q}g[q]}\right]$$

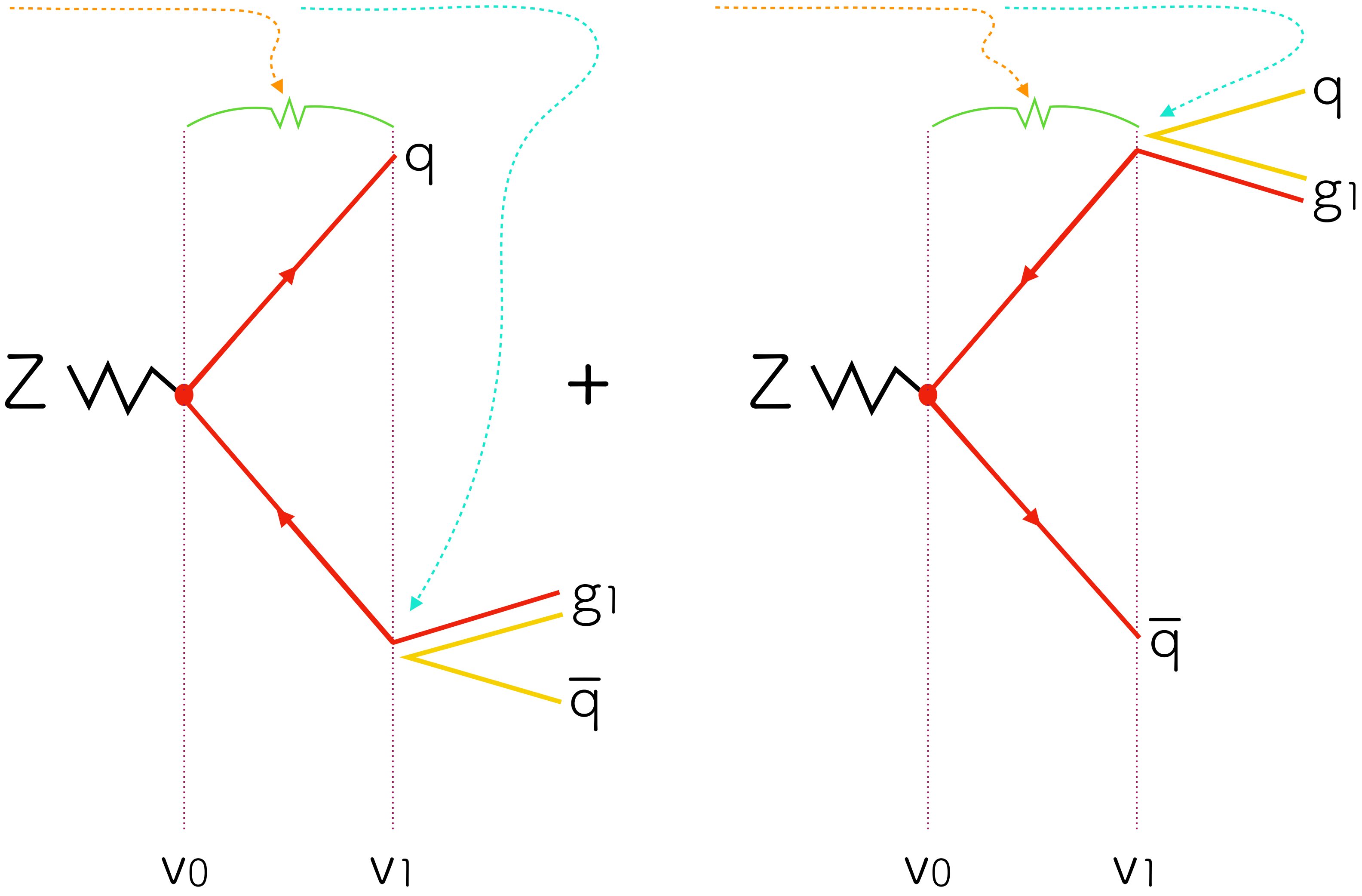
P[nothing in event emits in $v_0 \rightarrow v_1$]

$$\Delta\left(v_0, v_1\right) = \prod_{\substack{\text{dipole}\\ \text{ends } \tilde{i}}} \Delta_{\tilde{i}}\left(v_0, v_1\right)$$

Dipole showers: branching

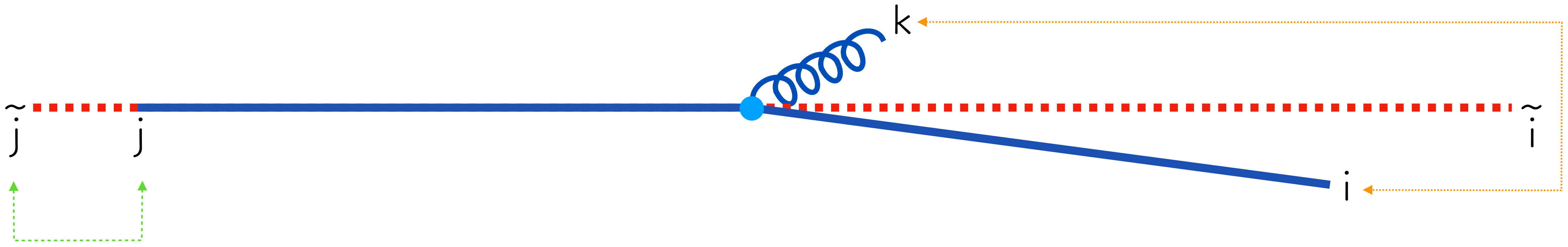
- Branching probability: evolution equation solved in terms of a Sudakov form factor

$$\Delta\left(v_0, v_1\right)\, d\mathcal{P}_{\bar{q}[q]\to\bar{q}g[q]} + \Delta\left(v_0, v_1\right)\, d\mathcal{P}_{q[\bar{q}]\to qg[\bar{q}]}$$



Dipole showers: local recoil

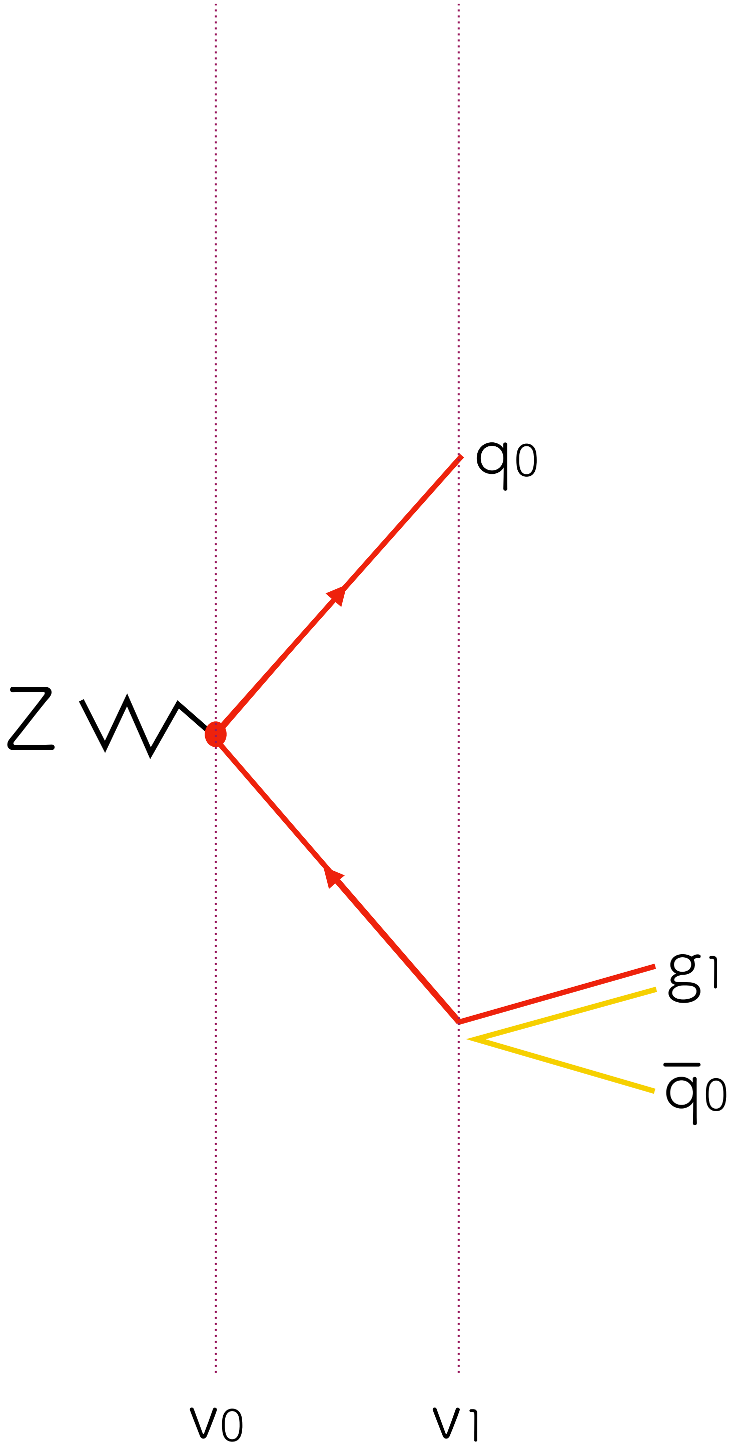
- Kinematic mapping: to ensure momentum conservation, the recoil is assigned locally (within the dipole)
- the *emitter* i takes the recoil of k in the $\tilde{i}\,\tilde{j}$ C.O.M. frame
- residual longitudinal recoil absorbed by the *spectator* j



$$\tilde{p}_i + \tilde{p}_j \xrightarrow{\tilde{p}_i \to p_i + p_k} p_i + p_j + p_k$$

$$\begin{aligned} p_i^\mu &= \tilde{z}\,\tilde{p}_i^\mu + y\,(1-\tilde{z})\,\tilde{p}_j^\mu + k_\perp \\ p_k^\mu &= (1-\tilde{z})\,\tilde{p}_i^\mu + y\,\tilde{z}\,\tilde{p}_j^\mu - k_\perp^\mu \\ p_j^\mu &= (1-y)\,\tilde{p}_j^\mu \end{aligned}$$

Dipole showers: iterate



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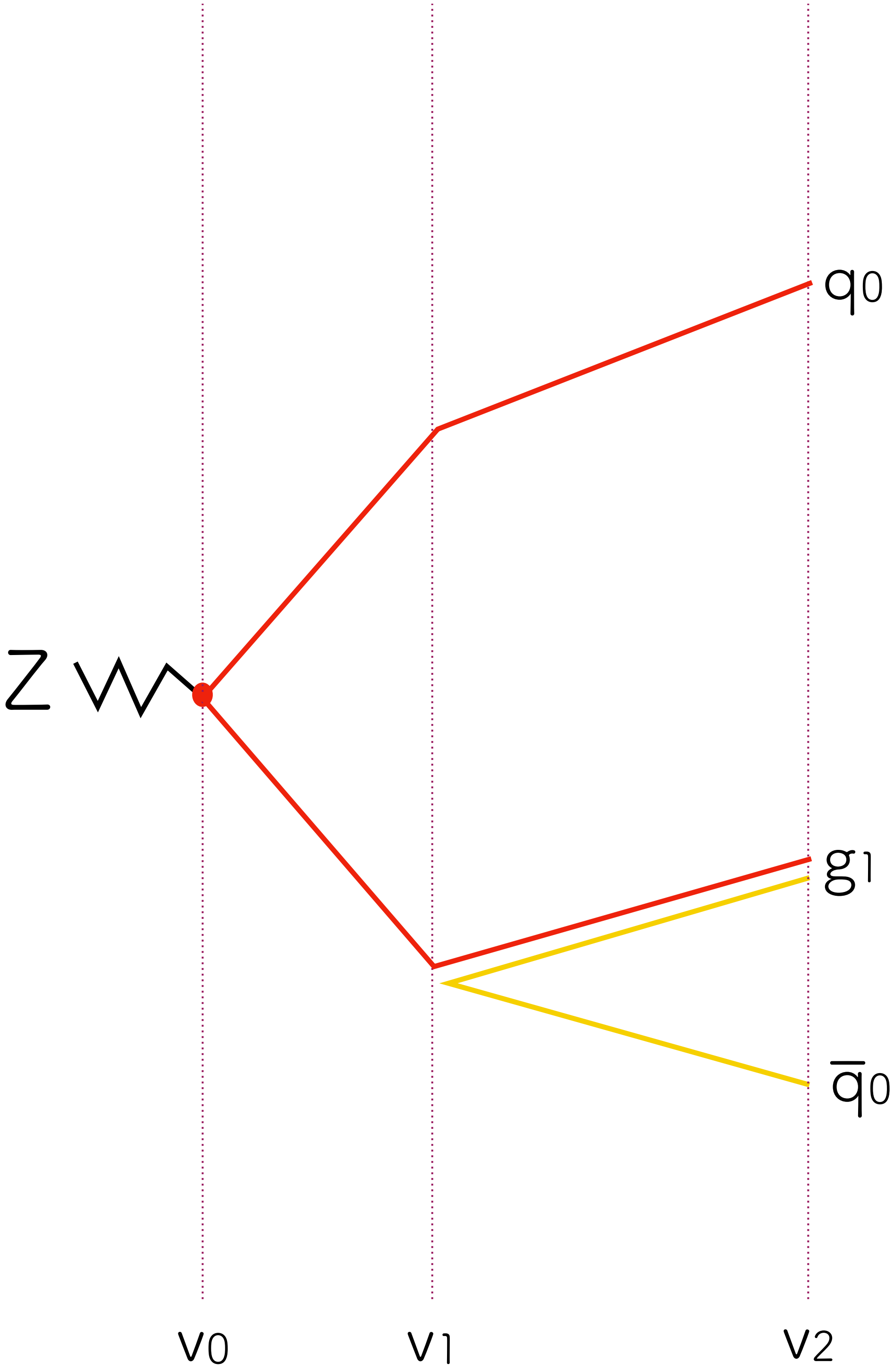

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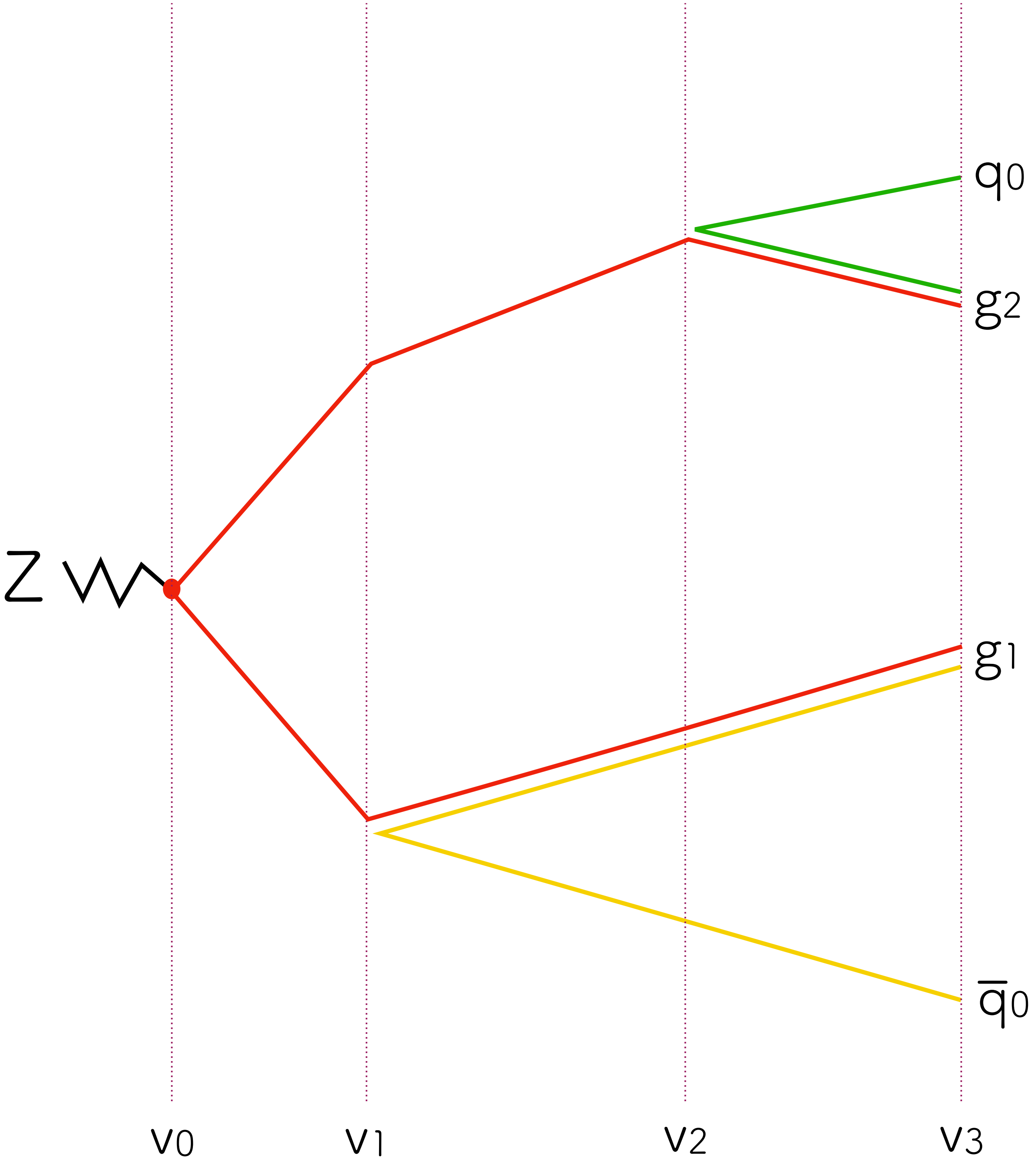

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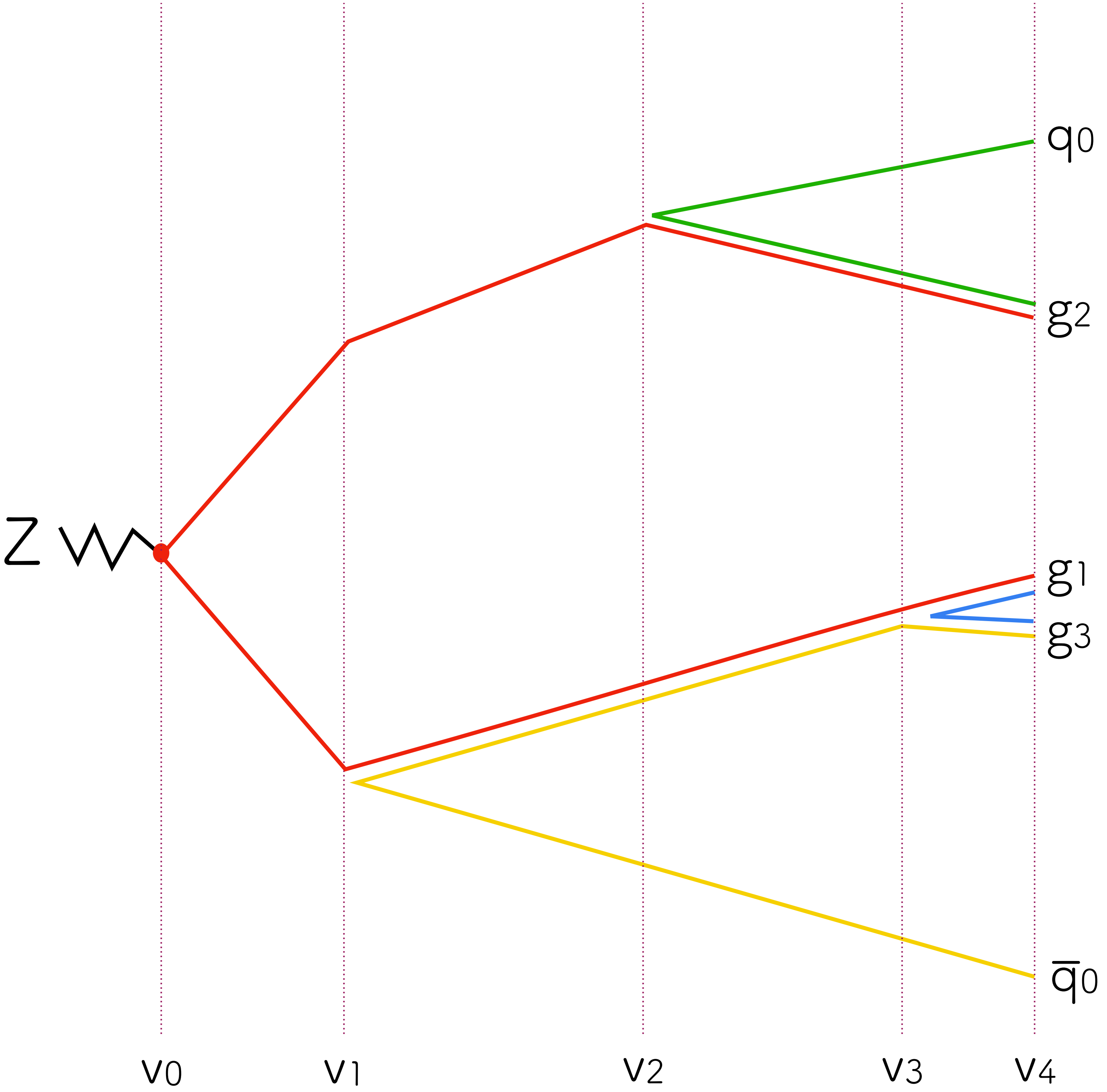

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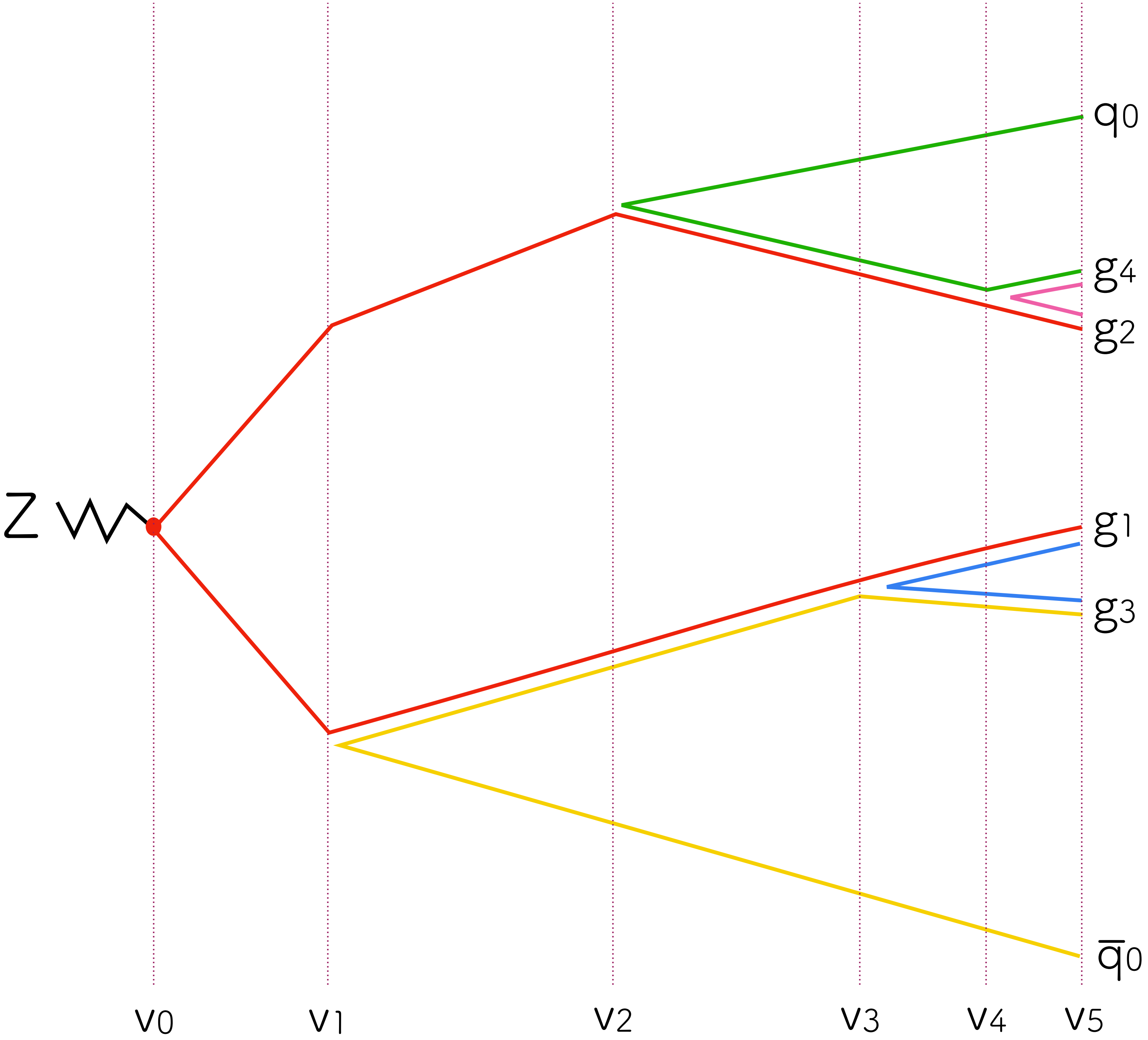

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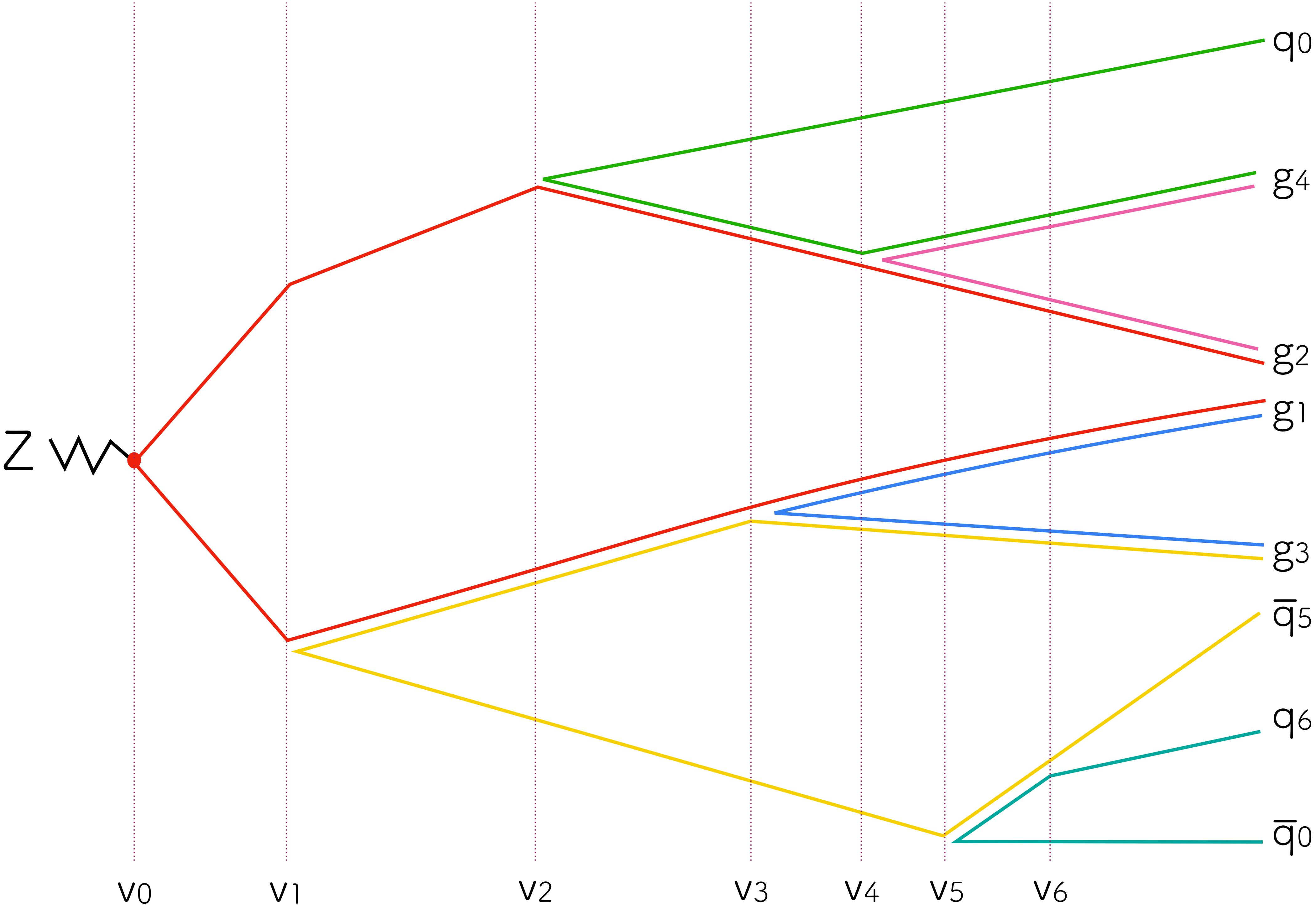

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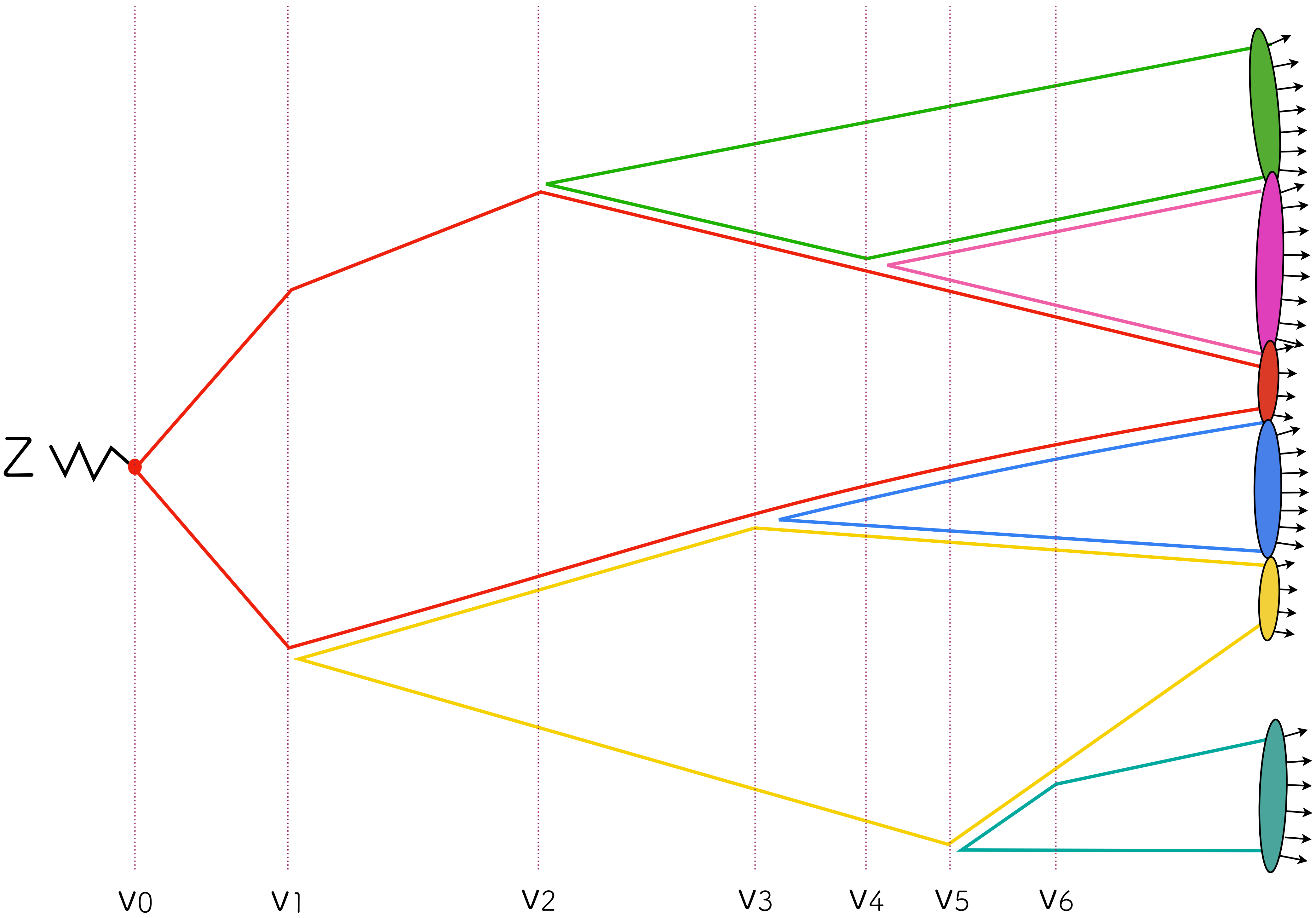
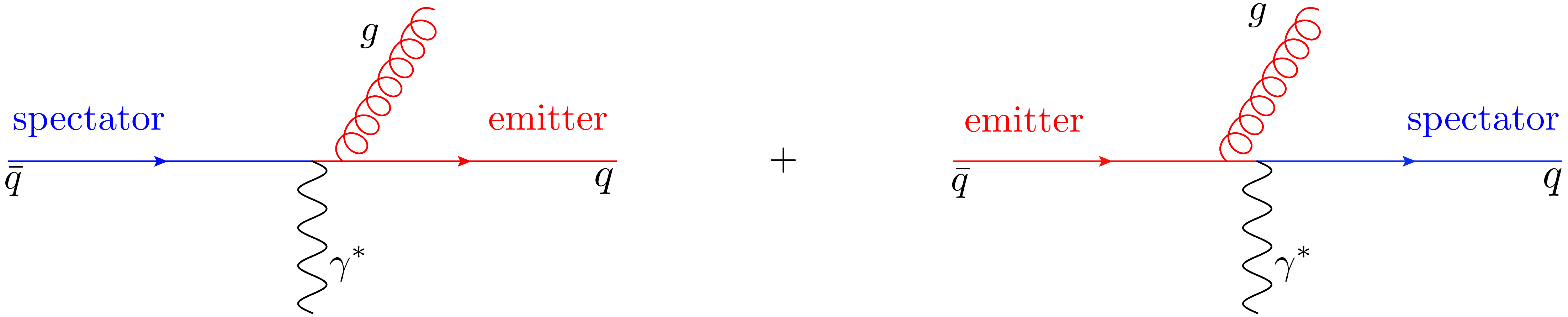


Single soft emission

Single emission: soft limit

- Both showers divide the dipole into two parts, at zero rapidity in the dipole's rest frame



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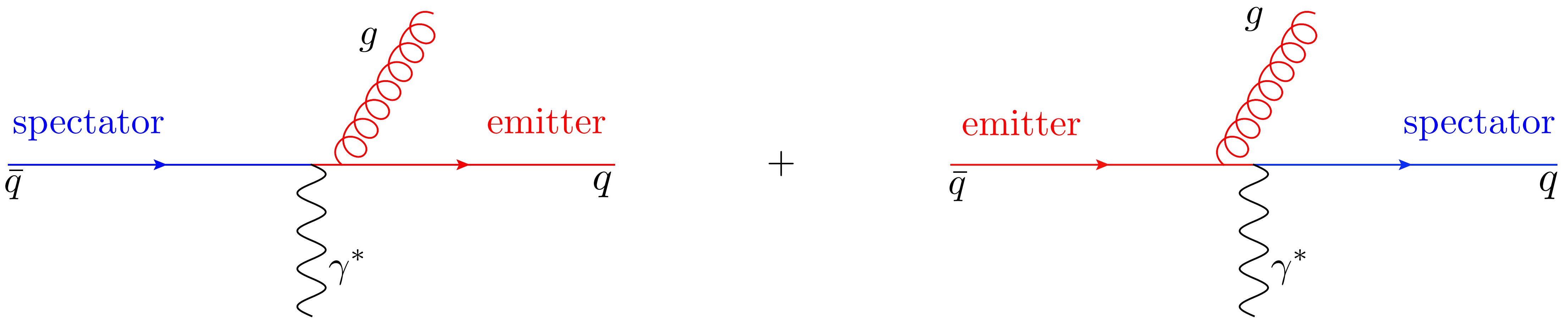


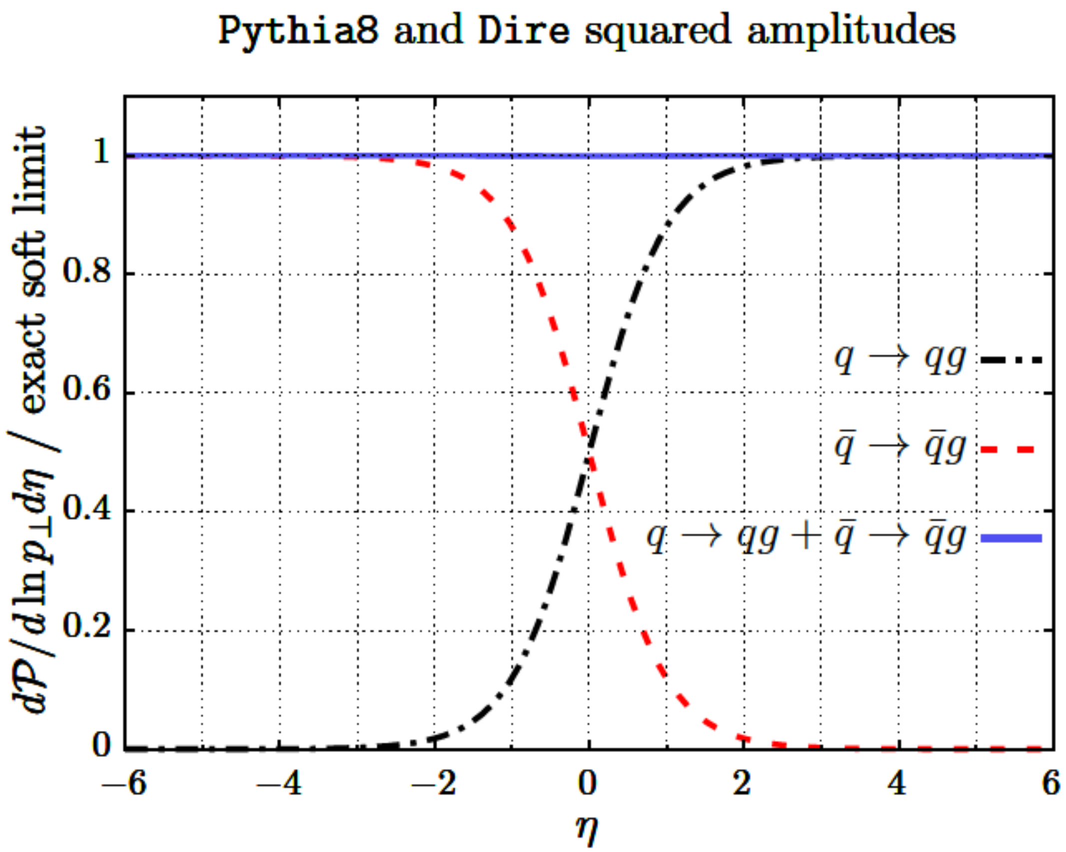
e.g. emission off the quark

Pythia

$$d\mathcal{P}_{q\to qg} = \frac{2\alpha_s(p^2_{\perp,\mathrm{evol}})C_F}{\pi}\frac{dp_\perp}{p_\perp}d\eta\left(\frac{e^{2\eta}}{1+e^{2\eta}}\right)$$

Dire

$$d\mathcal{P}_{q\to qg} = \frac{2\alpha_s(t)C_F}{\pi}\frac{dp_\perp}{p_\perp}d\eta\left(\frac{e^{2\eta}}{1+e^{2\eta}}\right)$$



Pythia8 and Dire squared amplitudes

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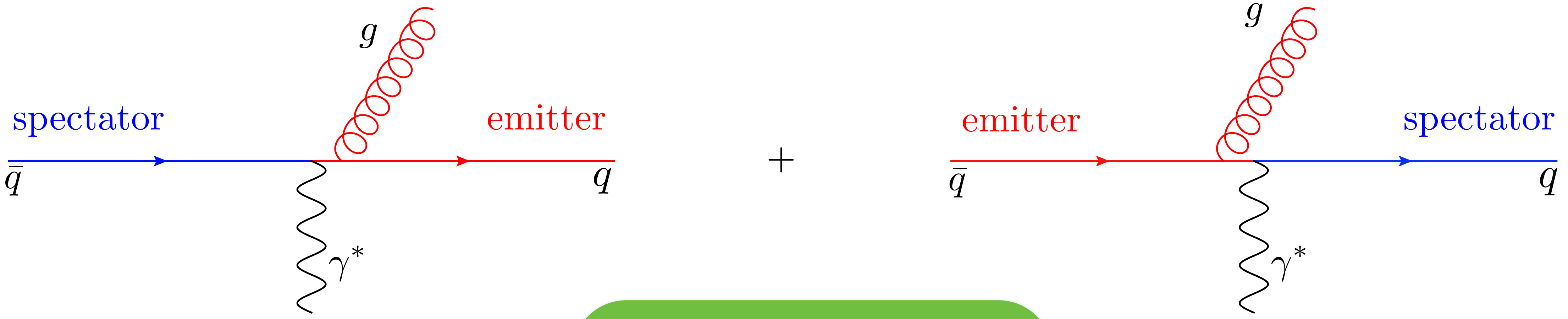

`e.g. emission off the quark`

Scale of the coupling should be the physical dipole k_t

Pythia

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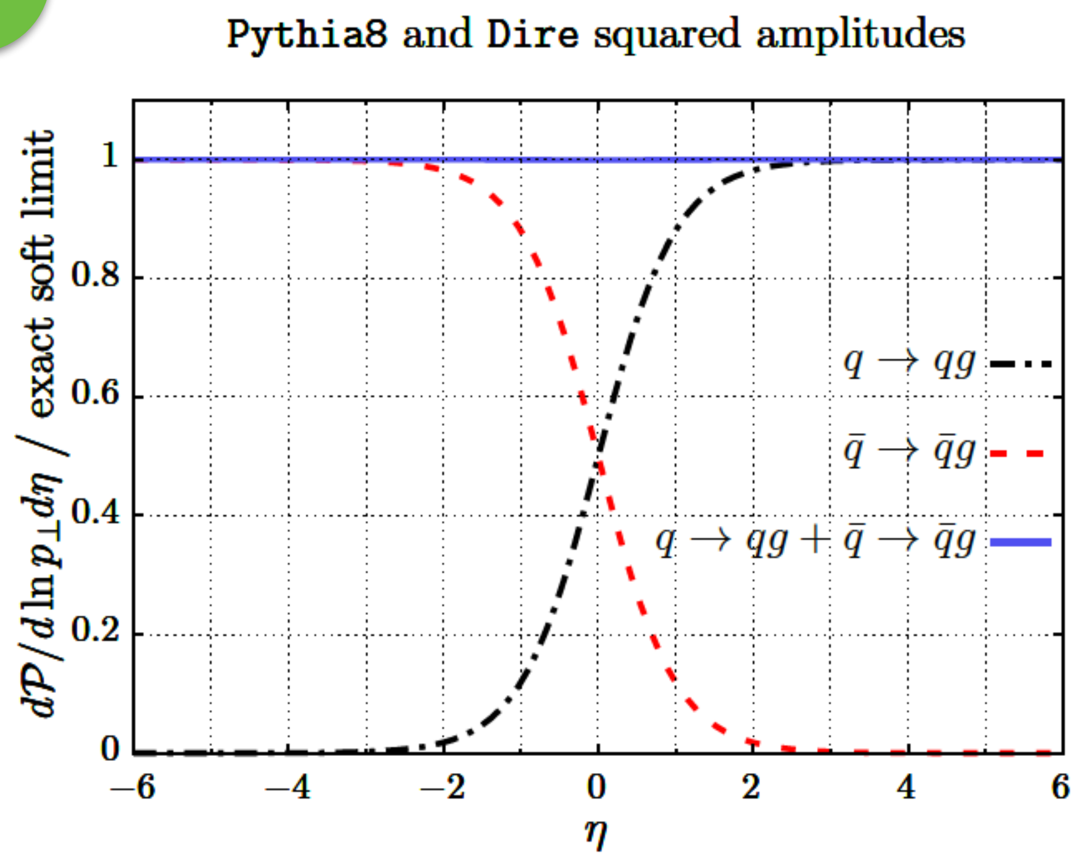
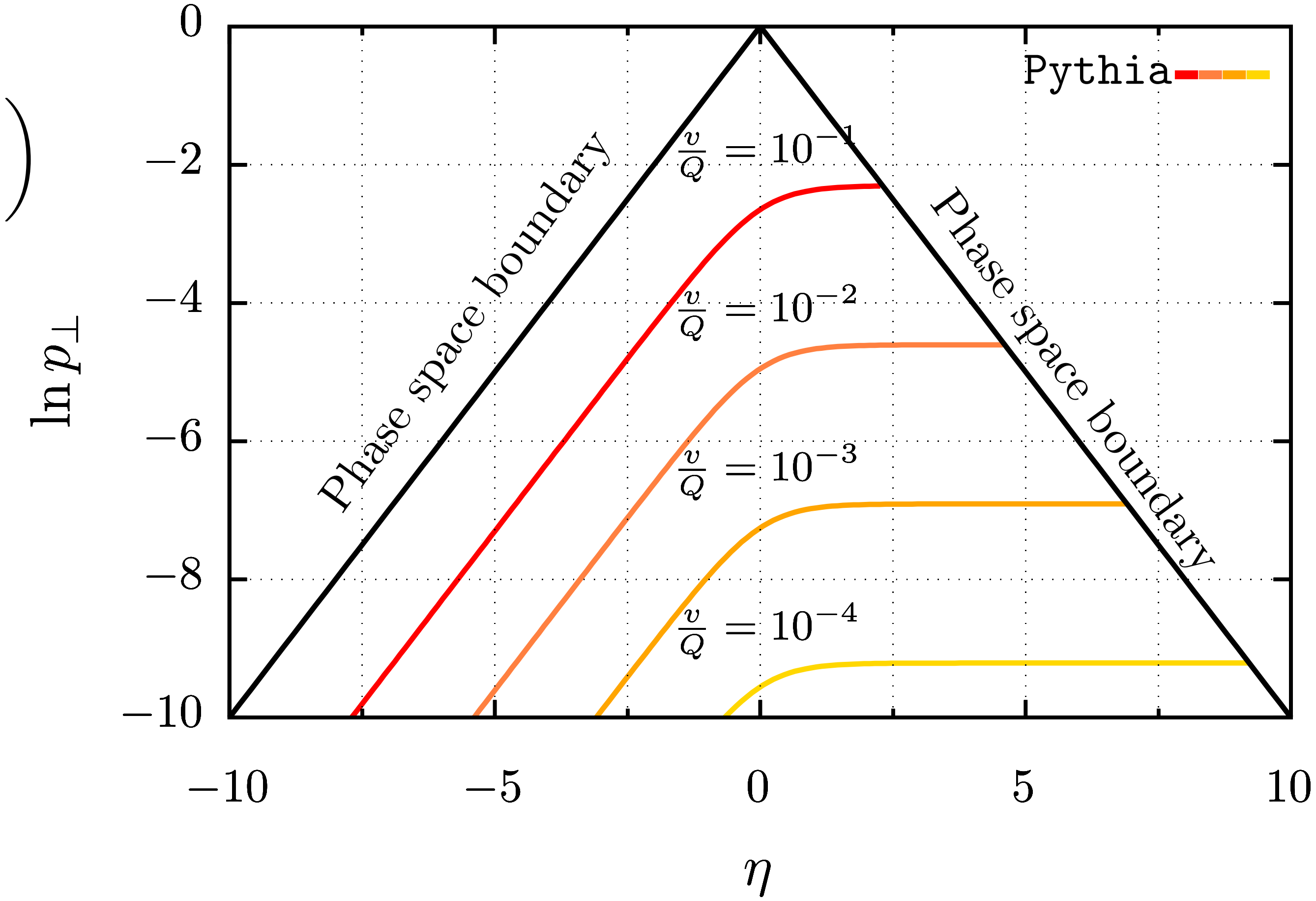

Pythia8 and Dire squared amplitudes

Single emission: soft limit

- `Pythia` case:

$$\eta = \frac{1}{2}\ln\left[\frac{(1-z)^2}{\rho^2_{\perp,\mathrm{evol}}} - 1\right], \qquad |p^2_\perp| = p^2_{\perp,\mathrm{evol}}\left(\frac{e^{2\eta}}{1+e^{2\eta}}\right)$$

$$d\mathcal{P}_{q\to qg} = \frac{2\alpha_s(p^2_{\perp,\mathrm{evol}})C_F}{\pi}\frac{dp_\perp}{p_\perp}d\eta\left(\frac{e^{2\eta}}{1+e^{2\eta}}\right)$$



Constant evolution variable contours in the Lund plane

- Correct matrix element for a single emission is reproduced up to running coupling effects

$$d\mathcal{P}_{q\to qg} + d\mathcal{P}_{\bar{q}\to\bar{q}g} = \frac{2\alpha_s C_F}{\pi}\frac{dp_\perp}{p_\perp}d\eta$$

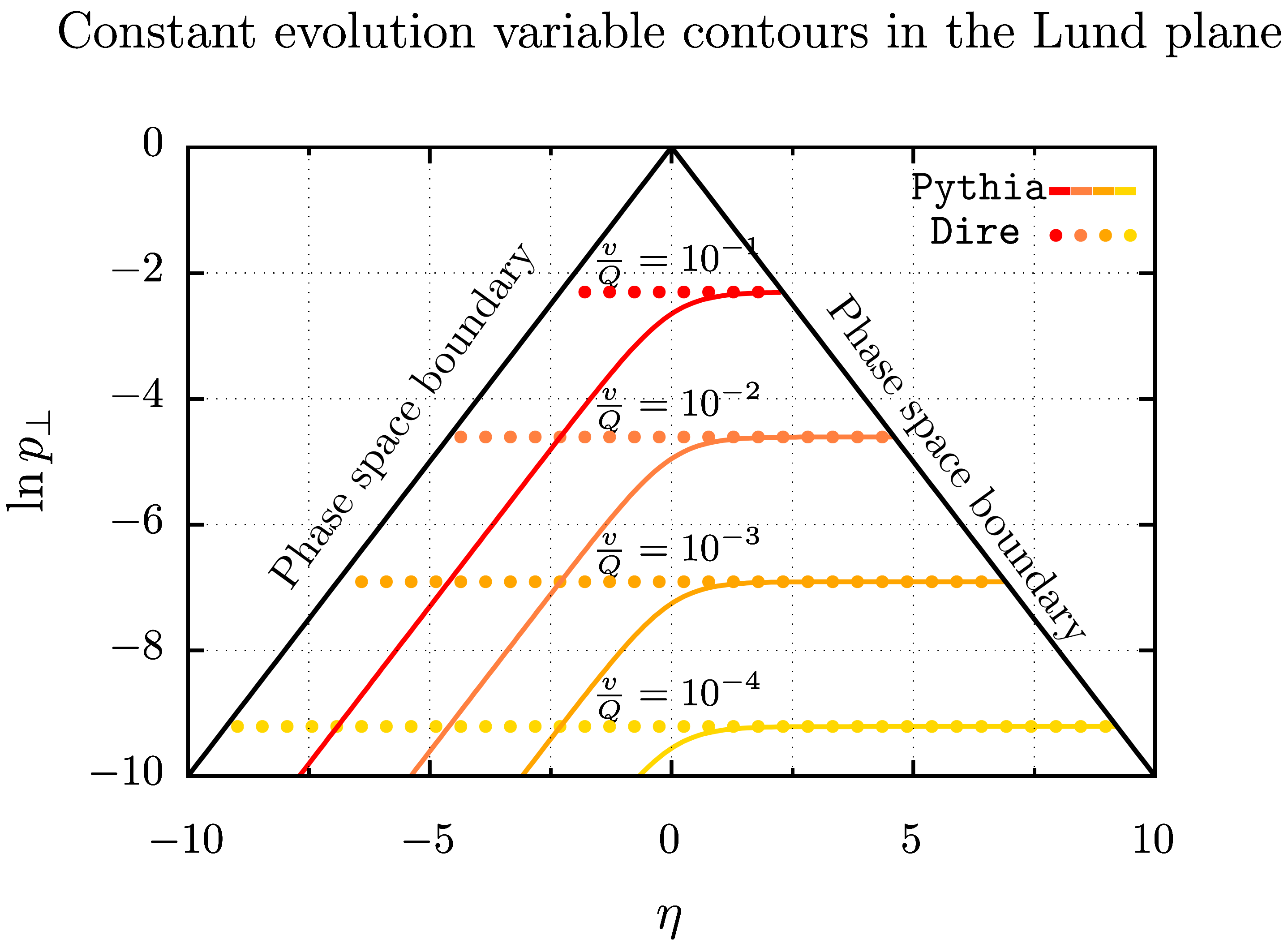
Not true anymore with running coupling in the soft-wide-angle region (NNLL effect)

- Non-zero (although suppressed) probability to have an emission with zero transverse momentum even if $p_{\perp,\mathrm{evol}} \neq 0$. This creates a new suppression mechanism in competition with the usual Sudakov suppression. In practice, unlikely to be of phenomenological interest

Single emission: soft limit

- `Dire` case:

$$\eta = \frac{1}{2} \ln \left[\frac{(1-z)^2}{\kappa^2} \right] , \qquad |p_\perp^2| = t$$



$$d\mathcal{P}_{q\to qg} = \frac{2\alpha_s(t) C_F}{\pi} \frac{dp_\perp}{p_\perp} d\eta \left(\frac{e^{2\eta}}{1+e^{2\eta}} \right)$$

- Correct matrix element for a single emission is reproduced including running coupling effects*

$$d\mathcal{P}_{q\to qg} + d\mathcal{P}_{\bar{q}\to \bar{q}g} = \frac{2\alpha_s(|p_\perp^2|) C_F}{\pi} \frac{dp_\perp}{p_\perp} d\eta$$

`* CMW scheme available both in Pythia and Dire`

Multiple soft emissions

Multiple emissions: soft limit

- We now consider two soft-collinear emissions (g$_1$ and g$_2$ with $v_1 > v_2$) in the limit where they are strongly ordered in angle. This approximation is relevant at NLL for all global, rIRC safe observables.
- From the resummation one expects that both gluons are emitted off the initial $q\bar{q}$ dipole with

$$dP_2 = \frac{C_F^2}{2!} \prod_{i=1,2} \left(\frac{2\alpha_s(p_{\perp,i}^2)}{\pi} \frac{dp_{\perp,i}}{p_{\perp,i}} d\eta_i \frac{d\phi_i}{2\pi} \right)$$

Multiple emissions: soft limit

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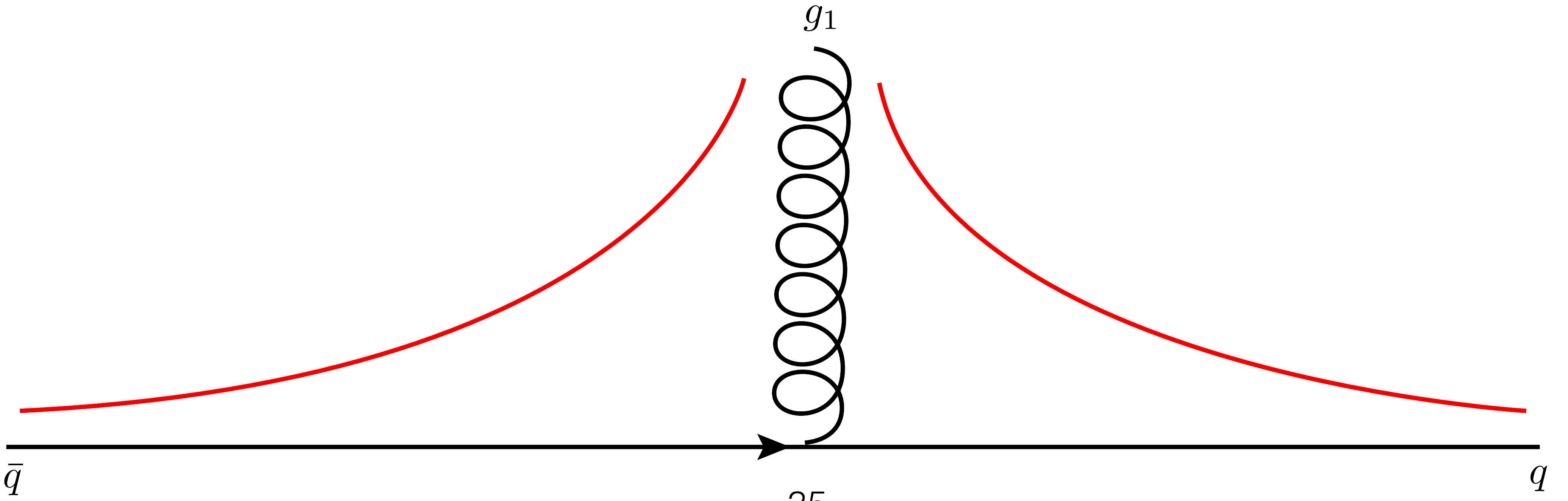
- Instead, the dipole-shower algorithm assigns the second emission to the first gluon in a portion of phase space in which it's collinear to the quarks: implications on logarithmic accuracy

`e.g.`

- `start with an emission g₁`



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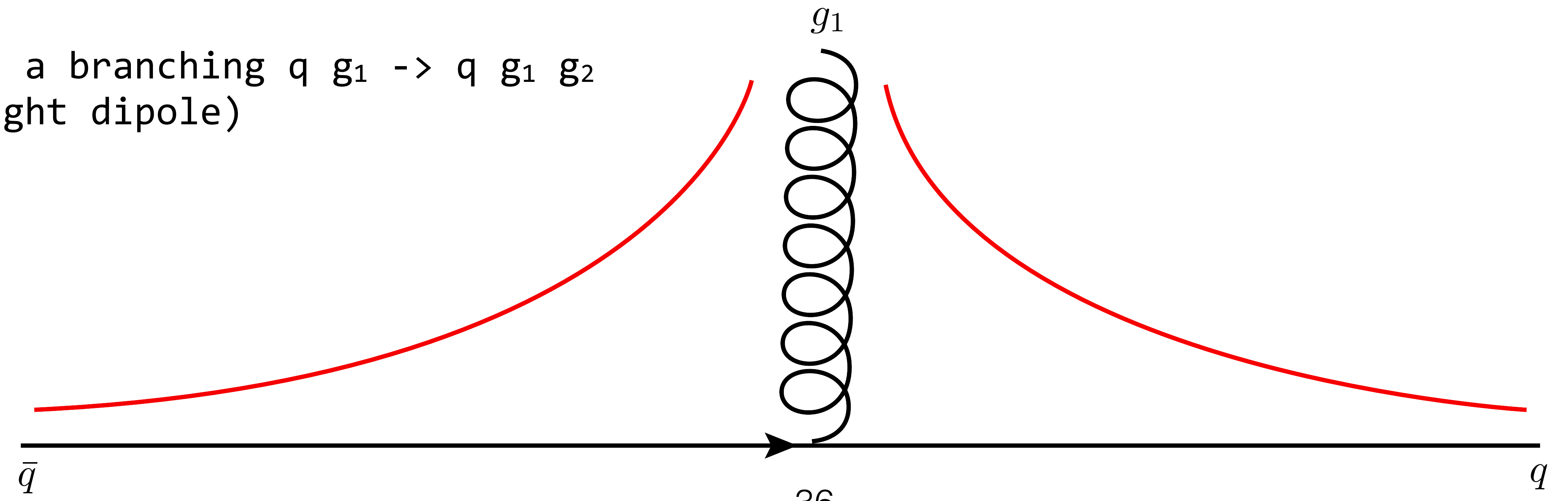
`e.g.`

- `start with an emission g₁`
- `add a branching q g₁ -> q g₁ g₂ (right dipole)`



Multiple emissions: soft limit

- We now consider two soft-collinear emissions (g_1 and g_2 with $v_1 > v_2$) in the limit where they are strongly ordered in angle. This approximation is relevant at NLL for all global, rIRC safe observables.
- From the resummation one expects that both gluons are emitted off the initial $q\bar{q}$ dipole with

$$dP_2 = \frac{C_F^2}{2!} \prod_{i=1,2} \left(\frac{2\alpha_s(p_{\perp,i}^2)}{\pi} \frac{dp_{\perp,i}}{p_{\perp,i}} d\eta_i \frac{d\phi_i}{2\pi} \right)$$

- Instead, the dipole-shower algorithm assigns the second emission to the first gluon in a portion of phase space in which it's collinear to the quarks: implications on logarithmic accuracy

e.g.

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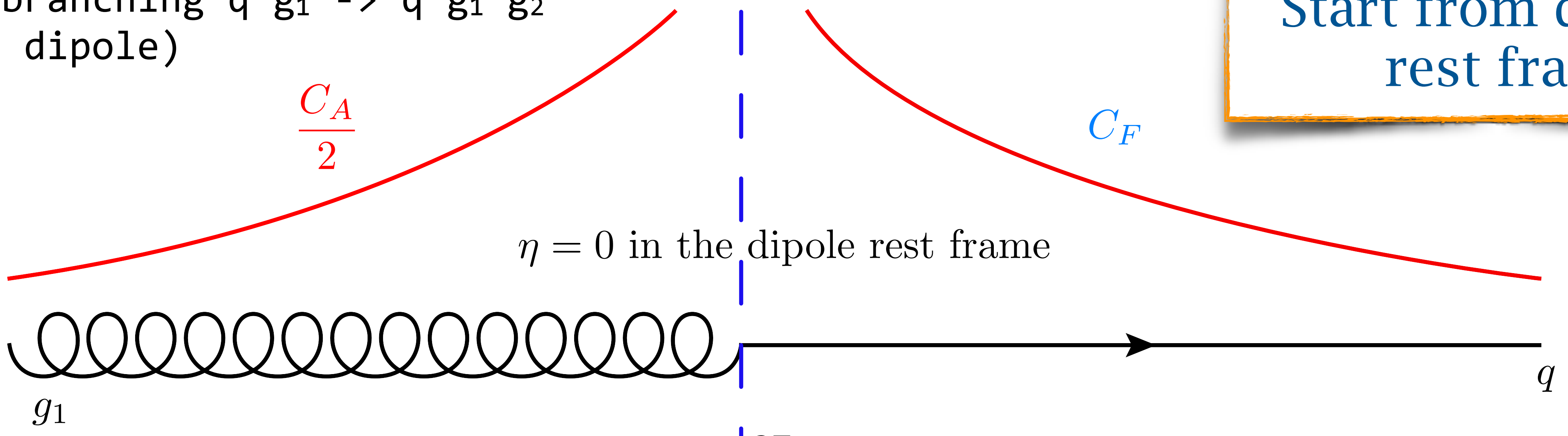

Start from dipole's rest frame

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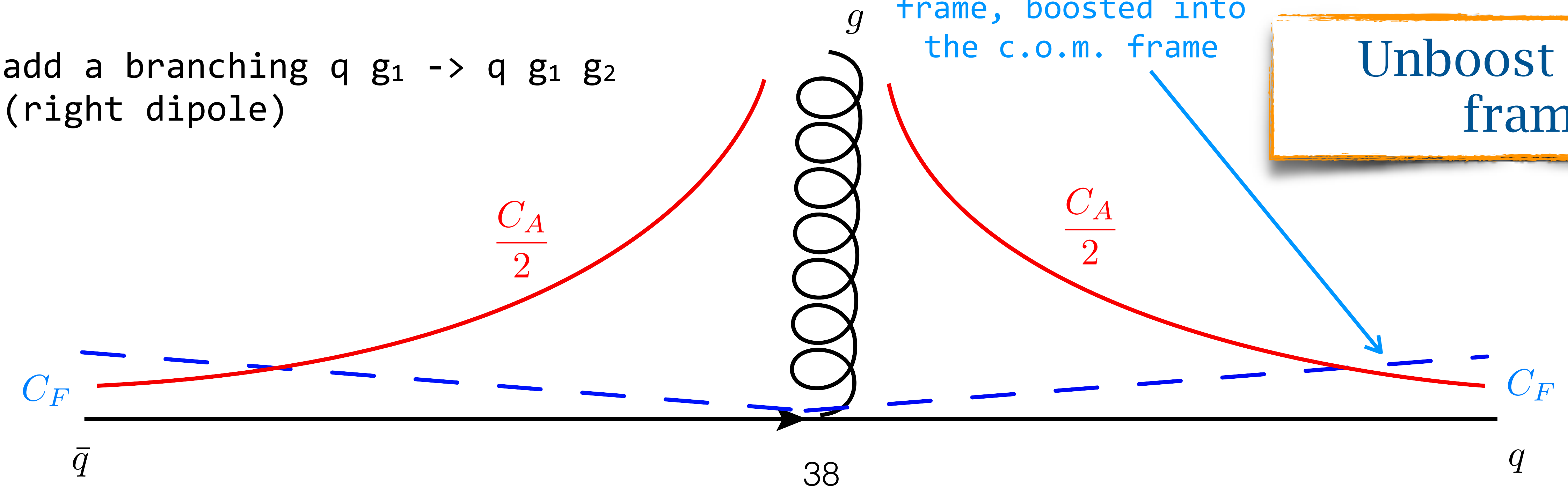


Line of zero rapidity in the dipole's rest frame, boosted into the c.o.m. frame

Unboost to lab frame

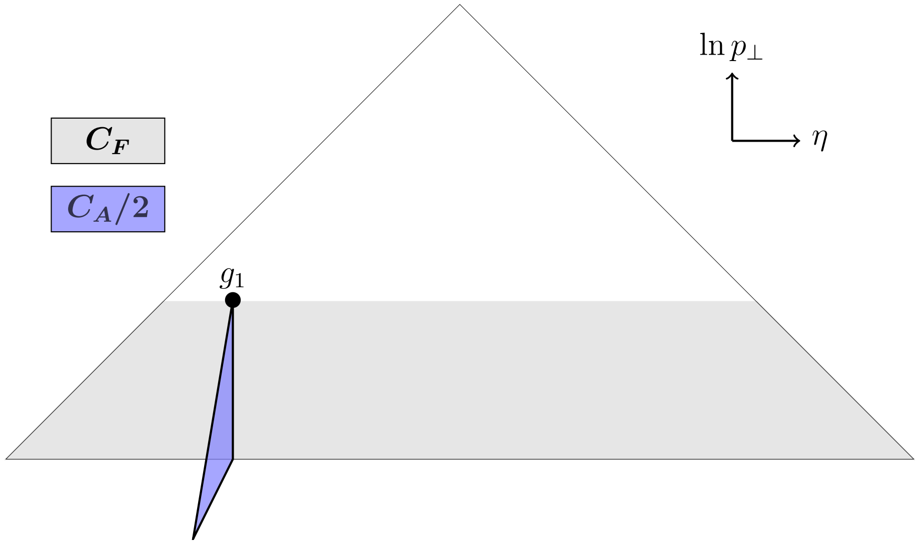
Double strong ordering

- Start by considering the limit where (in addition to angles) the ordering variable is strongly ordered, i.e. the kinematic of g$_1$ is not affected by the much softer g$_2$

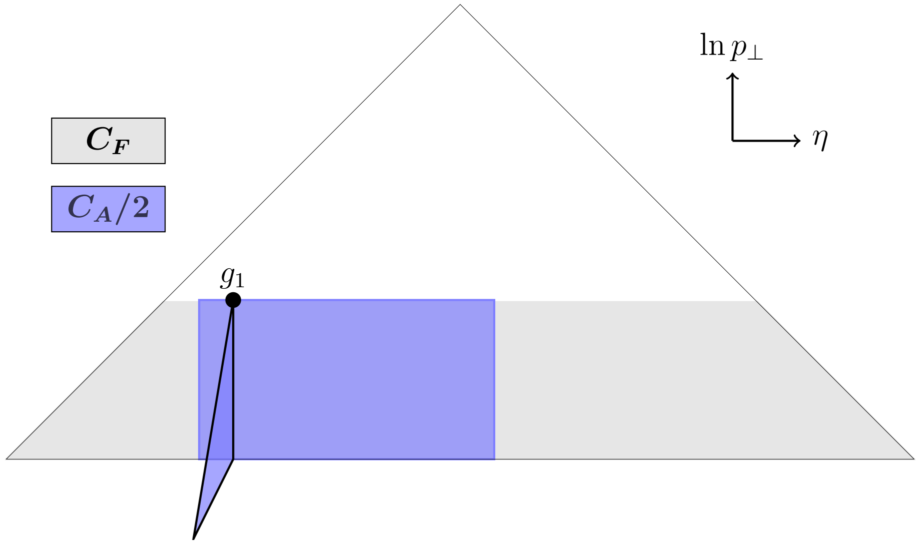
$$v_1 \gg v_2$$

- However, the colour charge for the second emission depends on the above partitioning

Correct radiation pattern



Dipole radiation pattern



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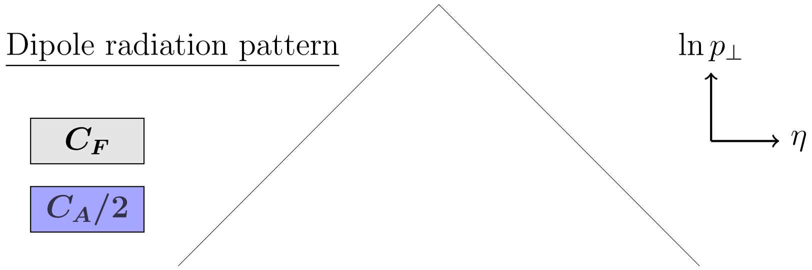

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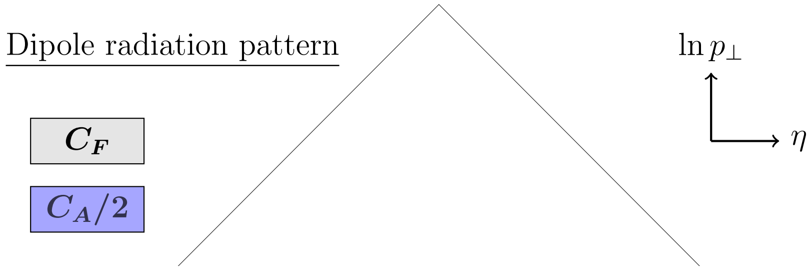

- This fact affects subleading colour corrections at different logarithmic orders

$$\Sigma(L) \equiv \int_0^{e^{-L}} \frac{dV}{\sigma_B} \frac{d\sigma}{dV} = \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots\right] + \mathcal{O}\left(\alpha_s e^{-L}\right)$$

$$\text{for an observable } V(p, \{\text{Born momenta}\}) \propto p_\perp^a e^{-|\eta_p| b}$$

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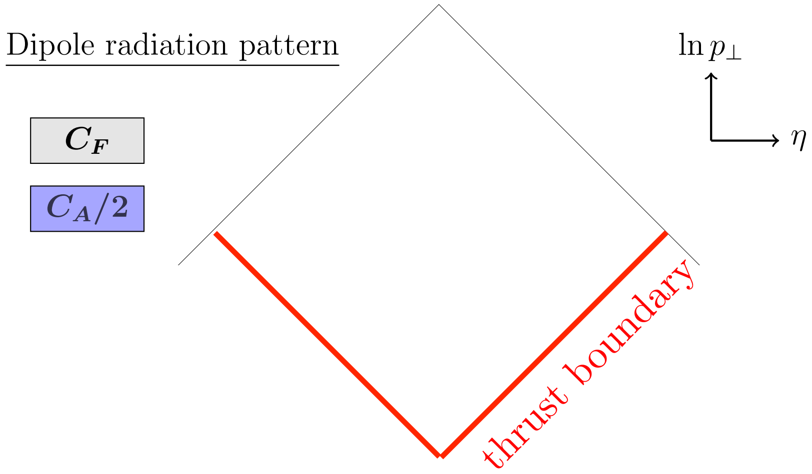

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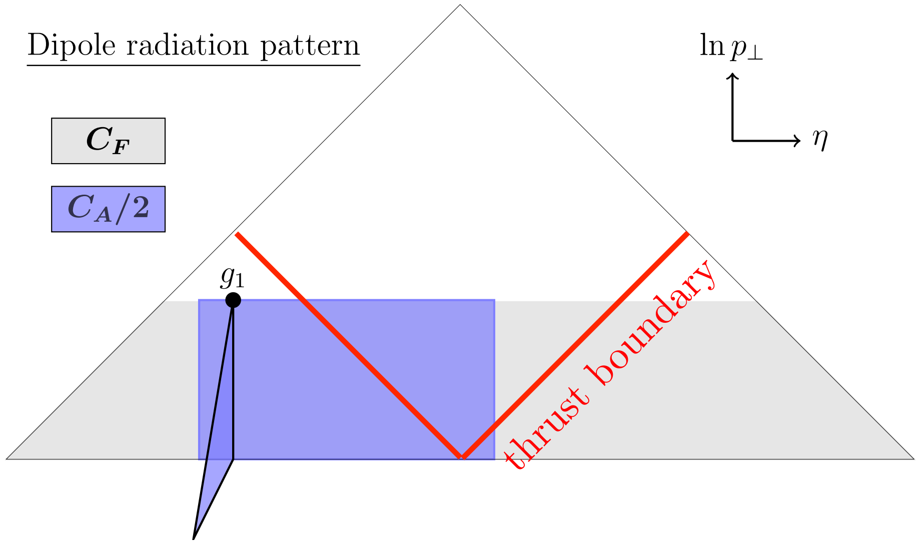

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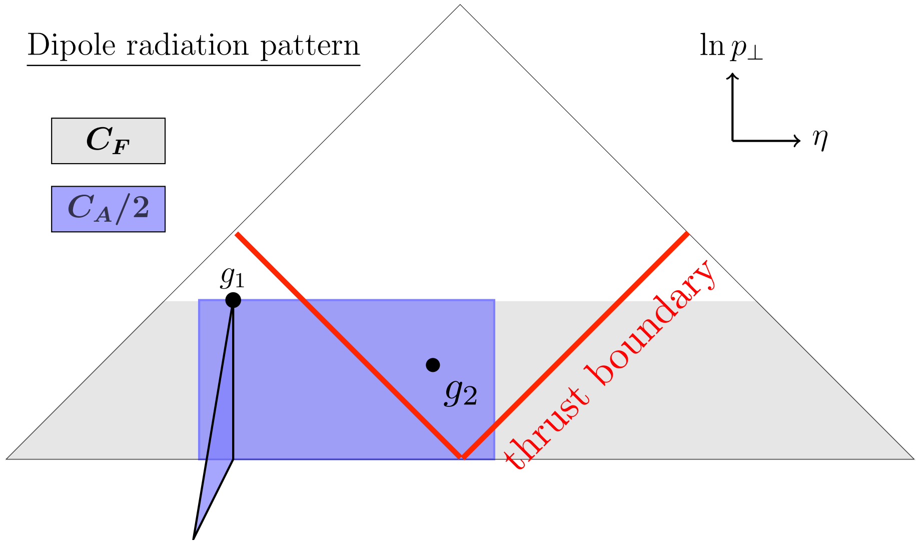

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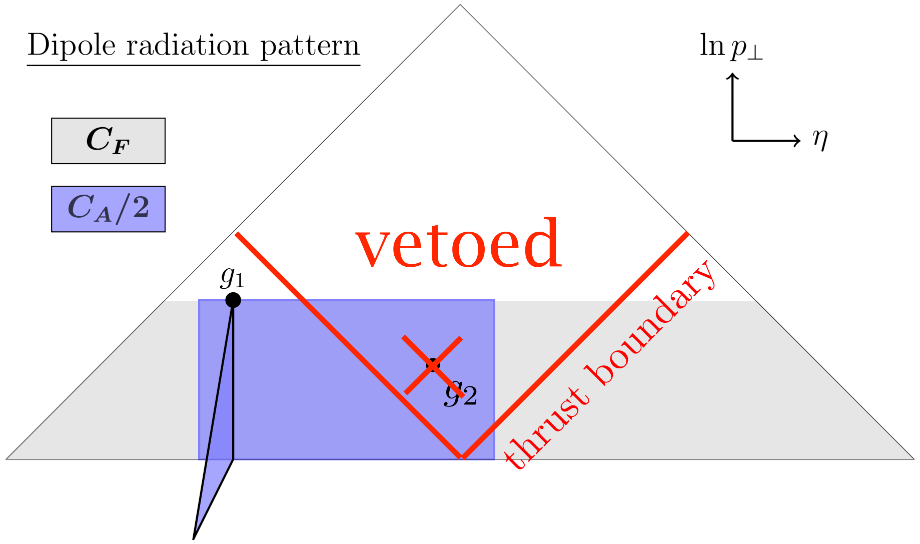

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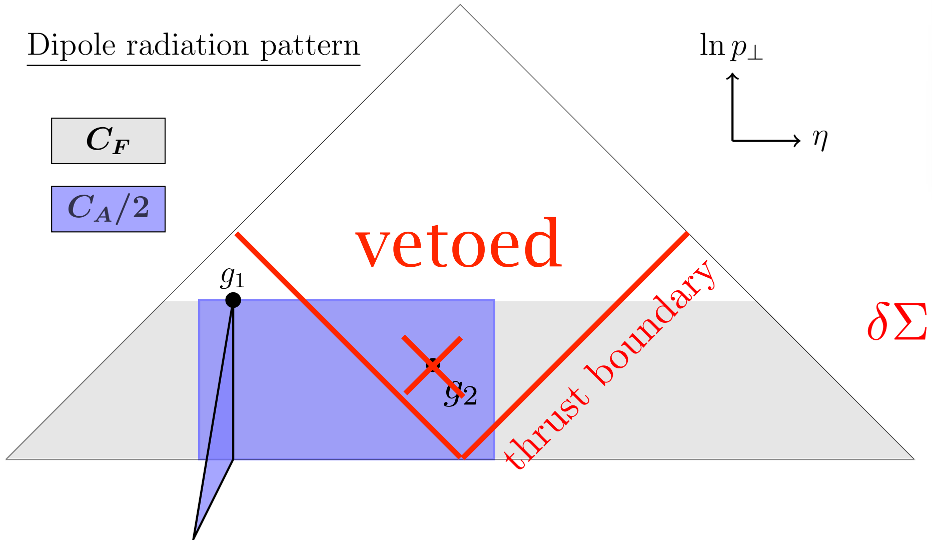

Difference between dipole and correct pattern

$$\delta\Sigma(L) = -\frac{1}{64}\left(\frac{2\alpha_s C_F}{\pi}\right)^2 L^4 \left(\frac{C_A}{2C_F} - 1\right)$$

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Single strong ordering: kinematics

- When the ordering variables are of the same order ($v_1 \gtrsim v_2$) the first emission g_1 is affected by the second (g_2) when this is far from g_1 in the lab frame
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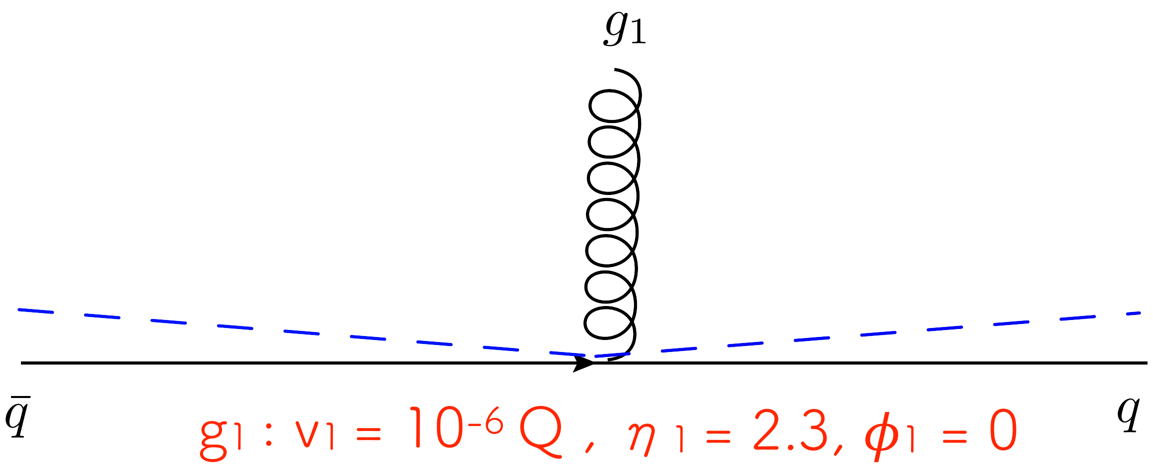
- Eventually reflected in the observables

`e.g.`

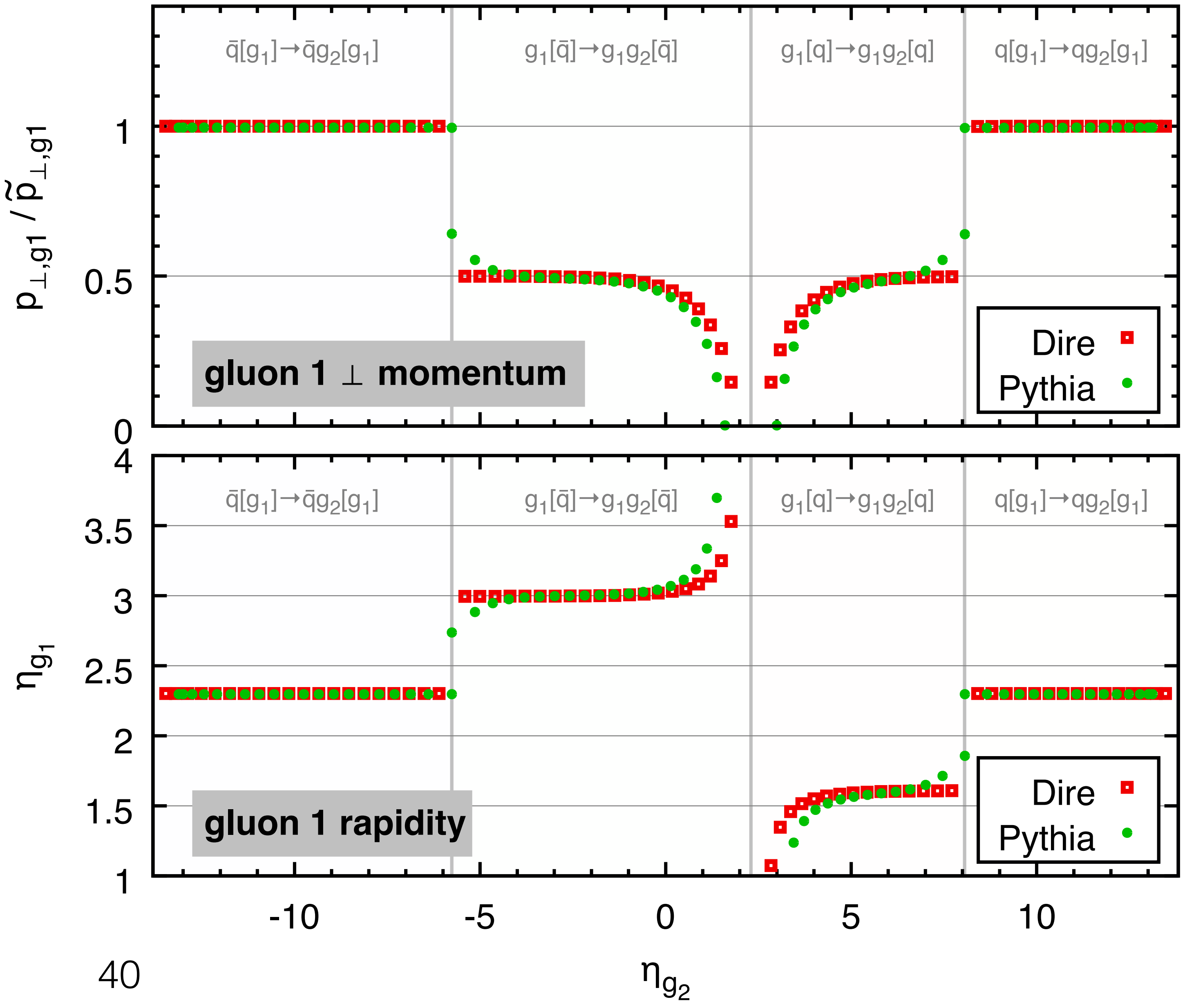
`● start with an emission g₁`



$g_1 : v_1 = 10^{-6}\,Q$, $\eta_1 = 2.3$, $\phi_1 = 0$

$g_2 : v_2 = 0.5\, v_1$, $\phi_2 = 0$, scan in η_2

impact of gluon-2 emission on gluon-1 momentum



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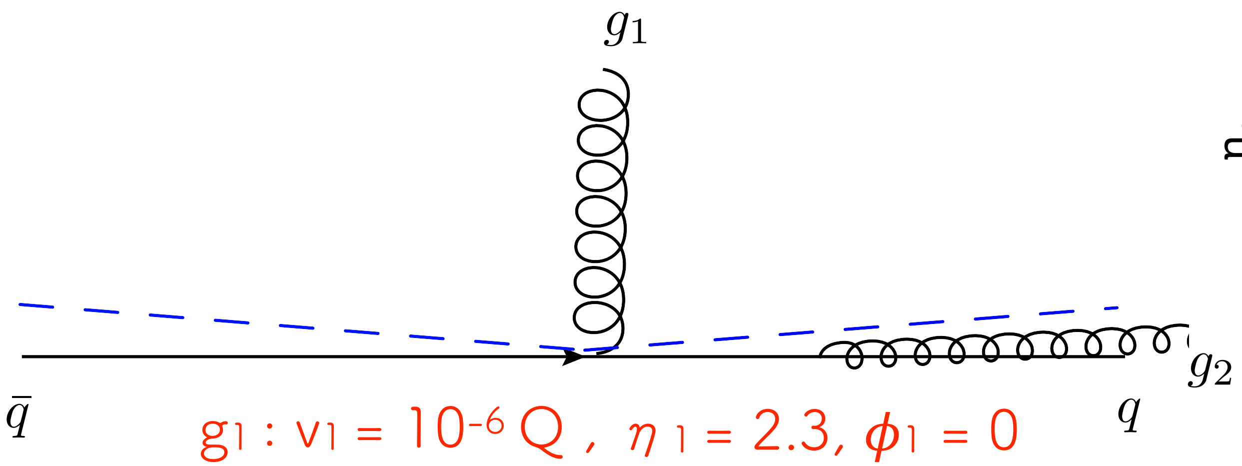

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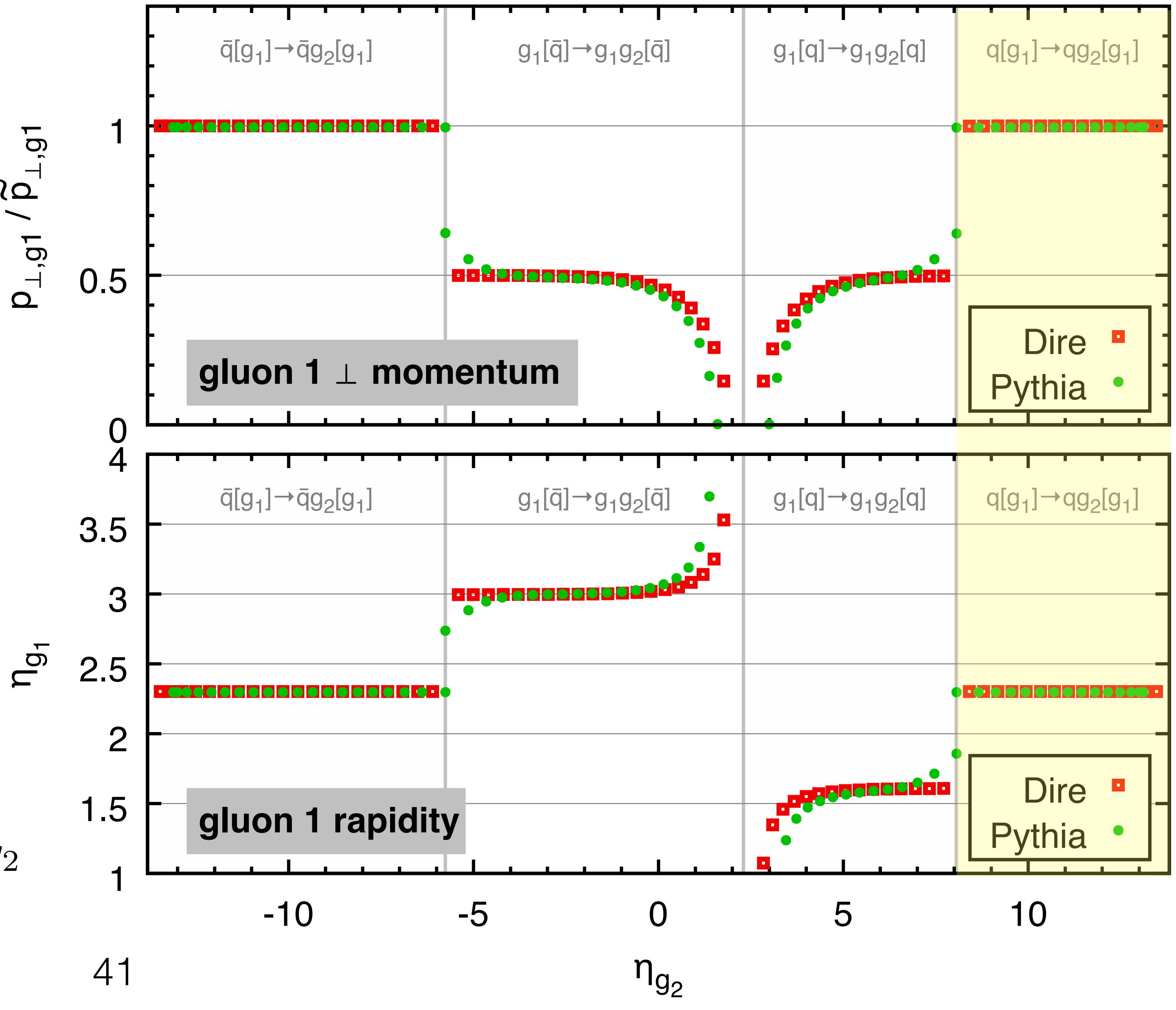

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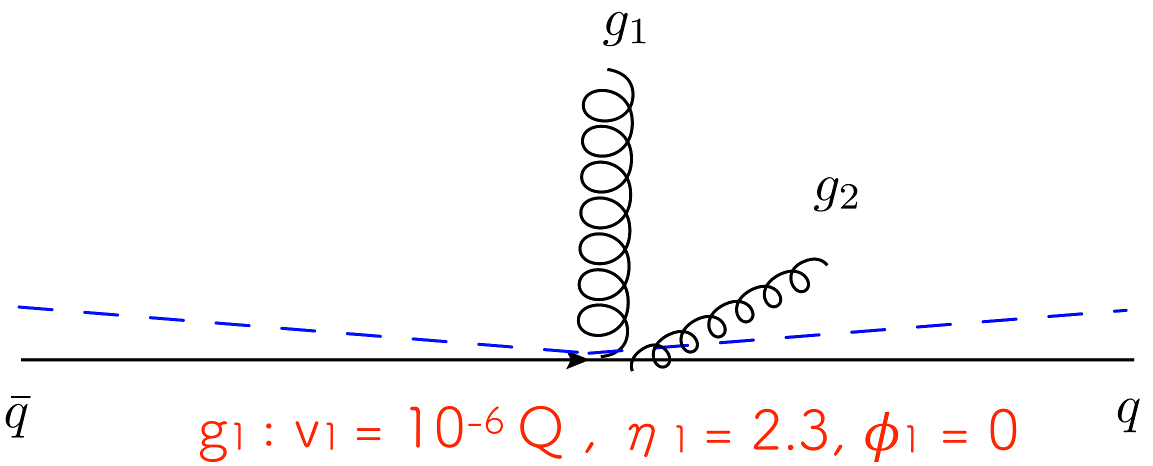

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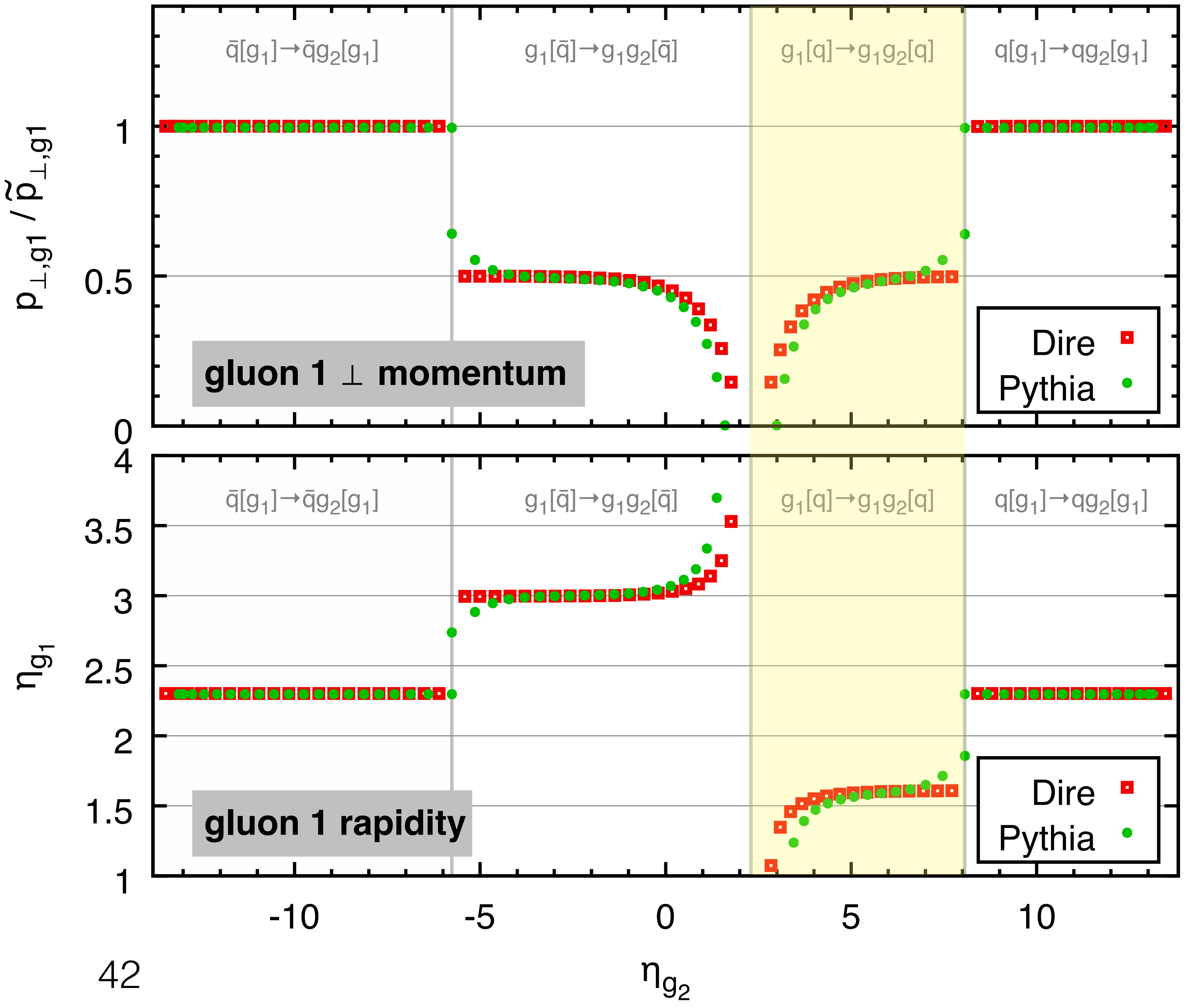

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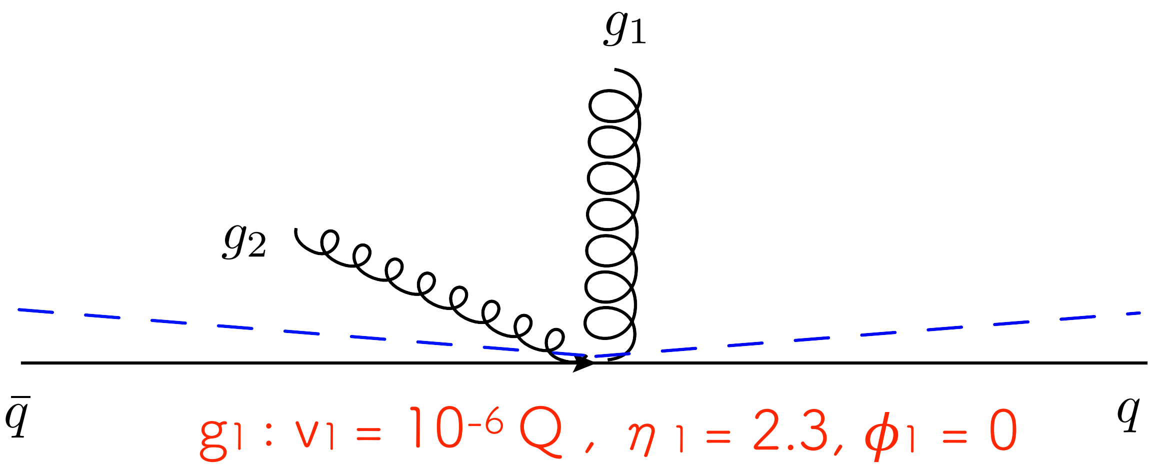

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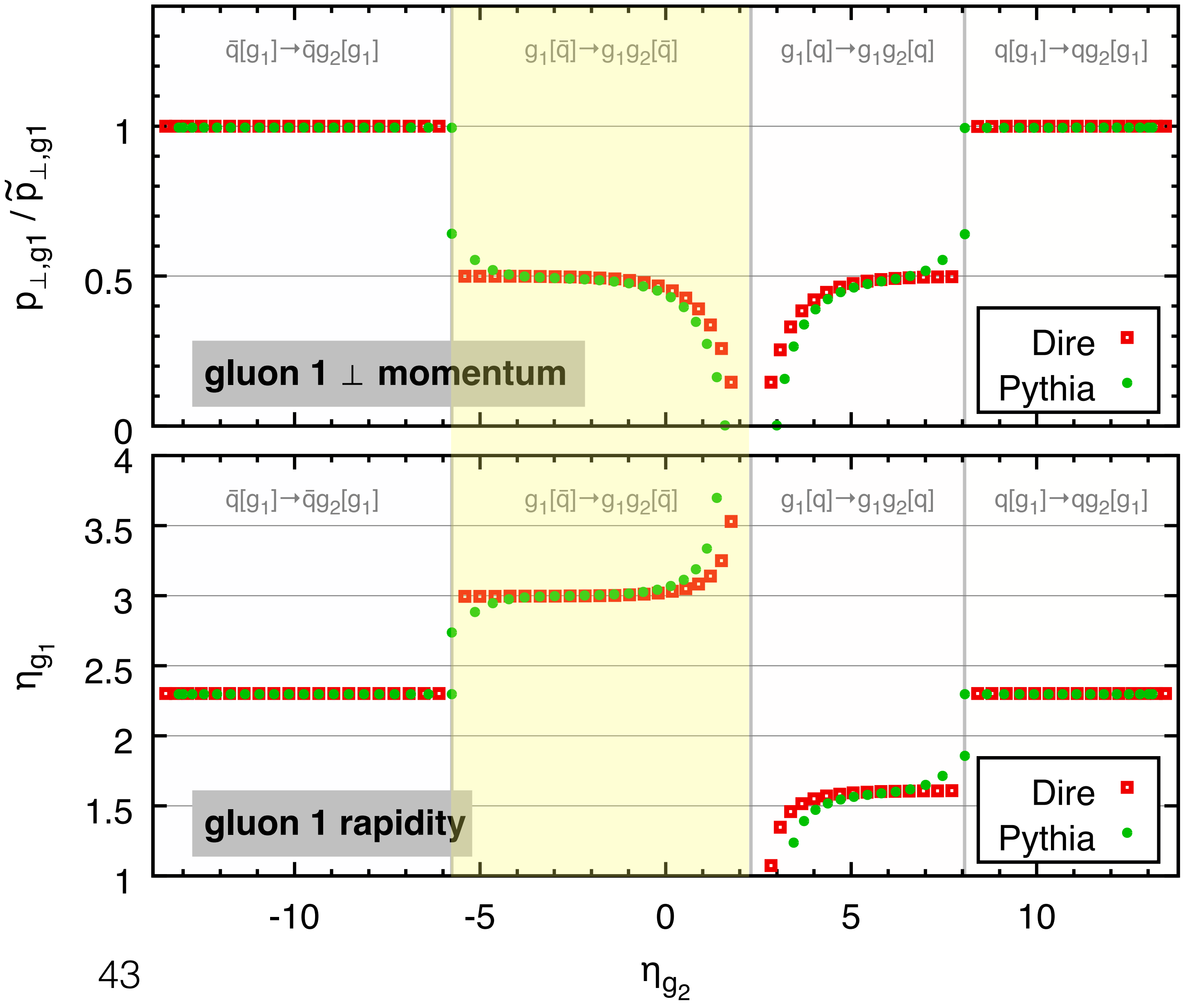

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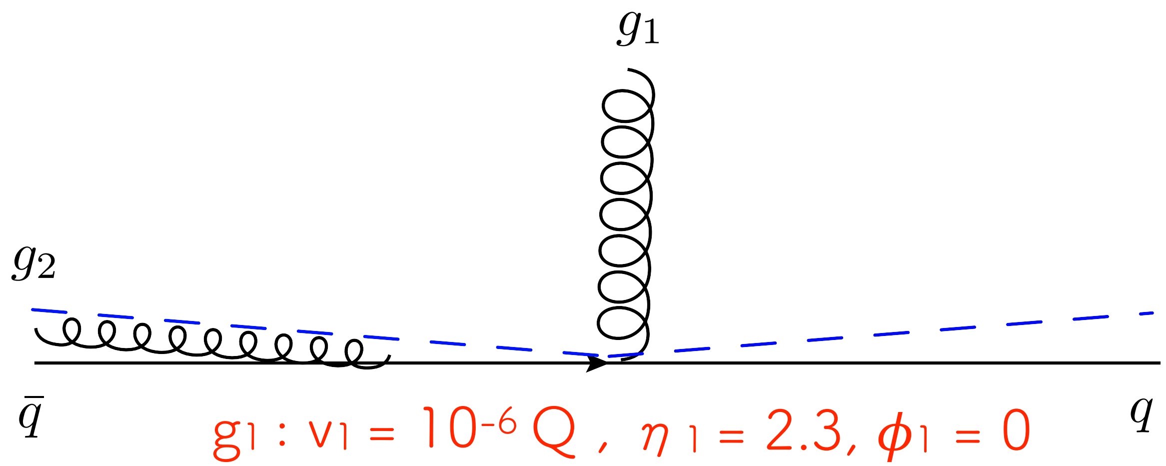

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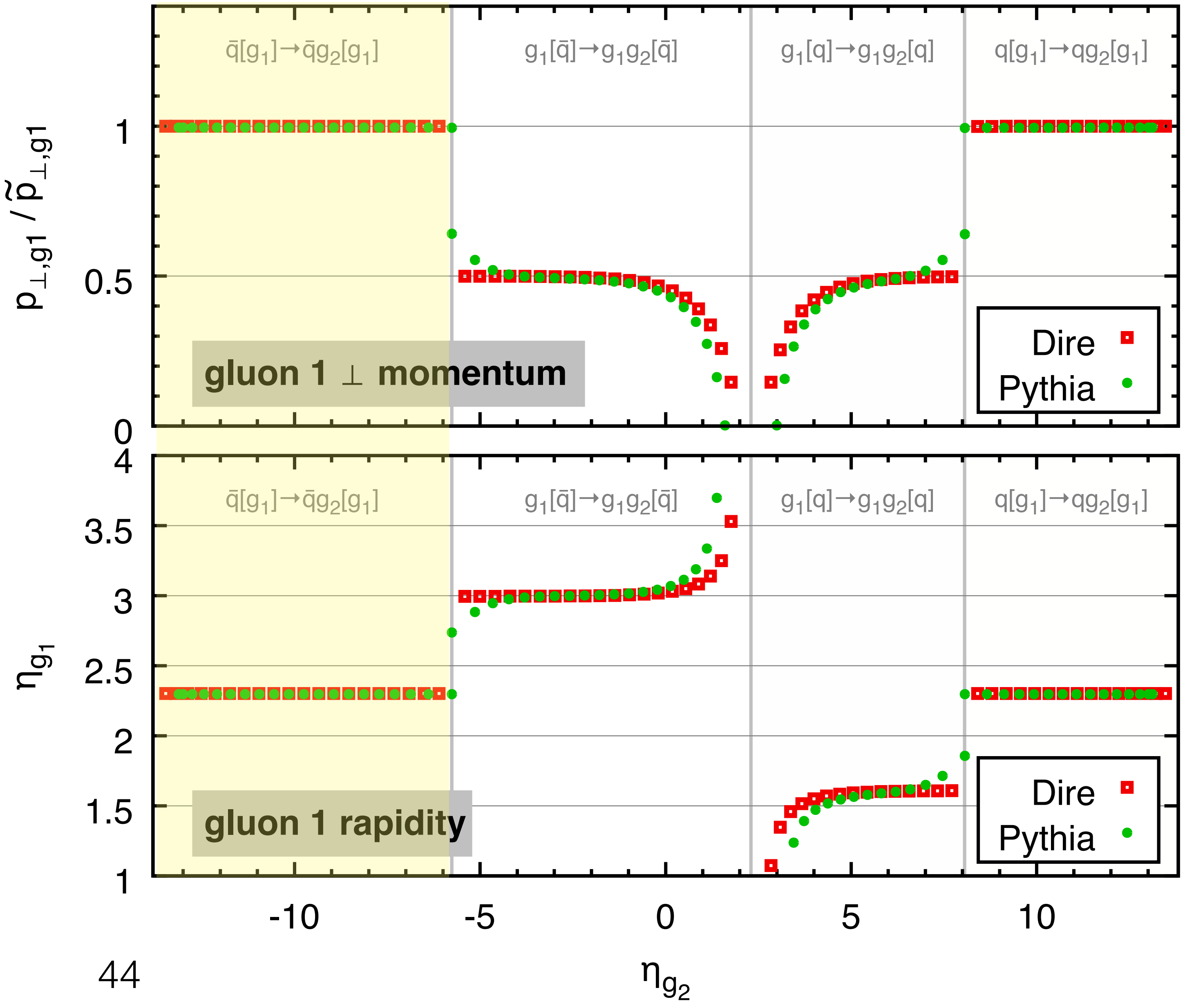
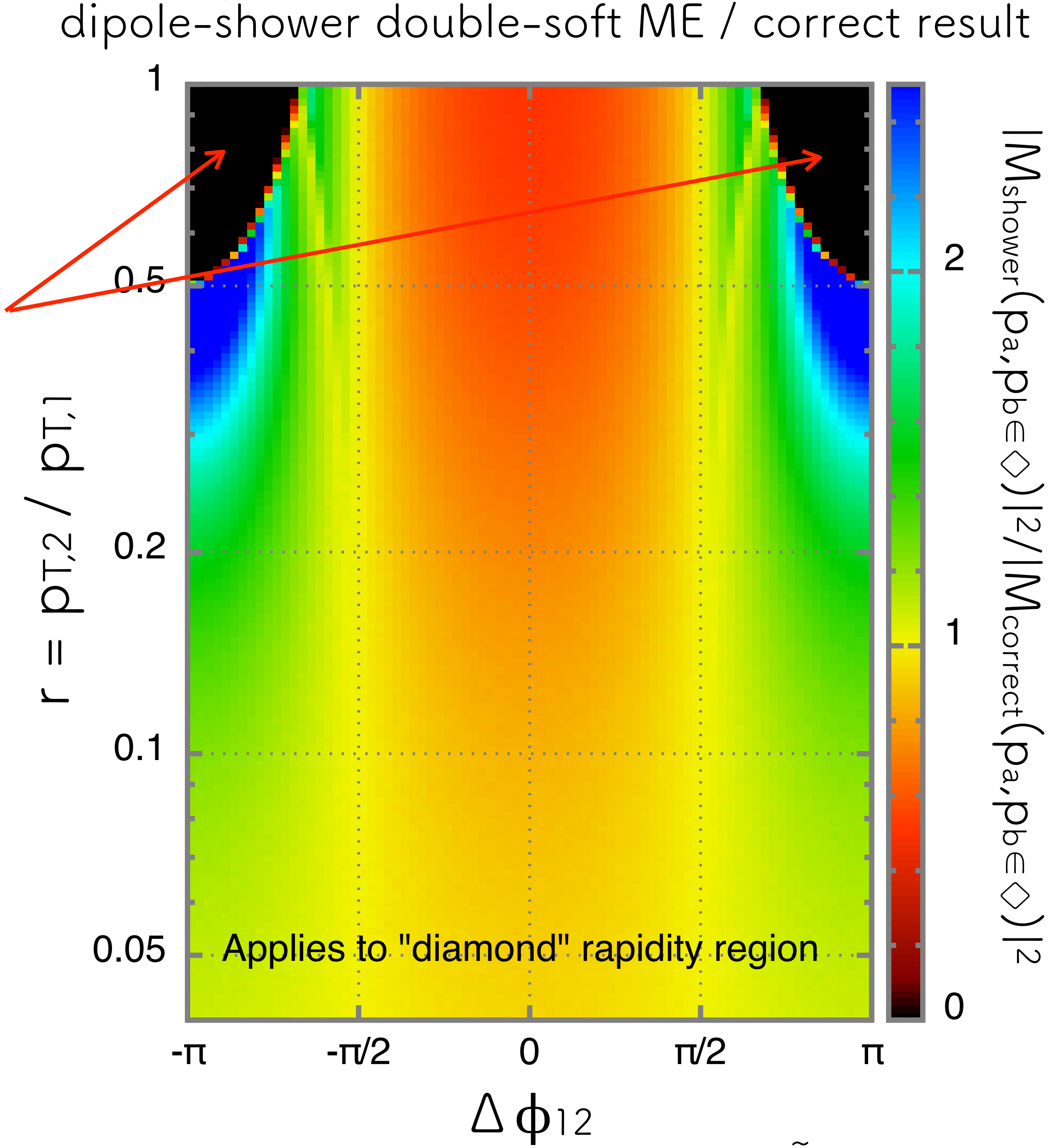


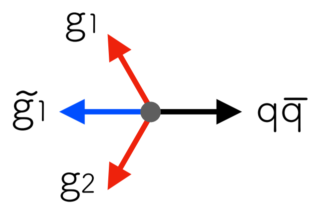
Single strong ordering: matrix element

- As a consequence, starting from second order, the effective matrix element differs from the NLL prediction
- Effects can be large for observables sensitive to exclusive regions of phase space
- This mechanism affects the pattern of subsequent real radiation, and virtual corrections, at all higher orders

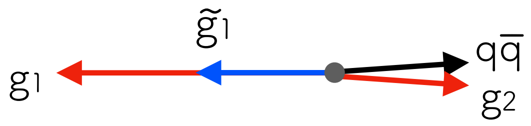
e.g. at α_s^2



➡E.g. $r = 1$, $|\Delta\phi| > \pm 2\pi/3$:



➡E.g. $|\Delta\phi| = \pm\pi$, $r > 0.5$:



Single strong ordering

- Occurs in a region relevant to NLL (leading colour) for all rIRC safe, global observables

e.g. 3-jet resolution in Cambridge algorithm

(angular ordered clustering of soft and/or collinear radiation)

$$\delta\Sigma^{(2\,\mathrm{emissions})}(L) = \left(C_F\frac{2\alpha_s}{\pi}\right)^2 \int_0^1 \frac{dv_1}{v_1}\int_{\ln v_1}^{\ln 1/v_1} d\eta_1 \int_0^{v_1}\frac{dv_2}{v_2}\int_{\ln v_2}^{\ln 1/v_2} d\eta_2 \int_0^{2\pi}\frac{d\phi_1}{2\pi}\int_0^{2\pi}\frac{d\phi_2}{2\pi}\times$$
$$\times\left[\Theta\left(e^{-L}-V(p_1^{\mathrm{shower}},p_2)\right)-\Theta\left(e^{-L}-V(p_1^{\mathrm{correct}},p_2)\right)\right]$$

$$V(p_1^{\mathrm{correct}},p_2)=v_1 \qquad V(p_1^{\mathrm{shower}},p_2)=\max\left(v_2,\,\sqrt{v_1^2+v_2^2-2v_1v_2\cos\phi_{12}}\right)$$

$$\delta\Sigma^{\mathrm{cam}}(L) = -0.18277\,\bar{\alpha}^2L^2+\mathcal{O}\left(\bar{\alpha}^2L\right)$$

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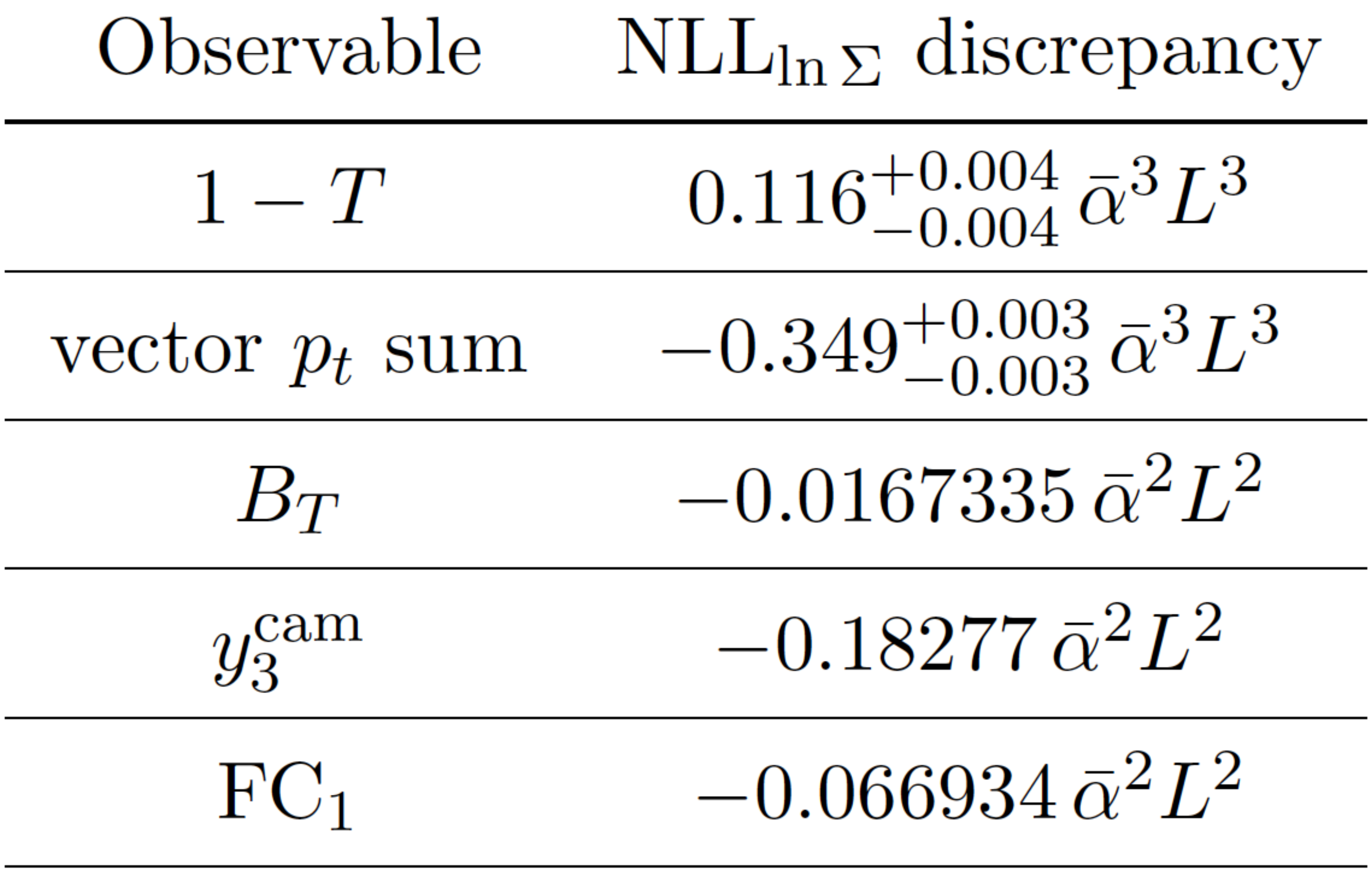
e.g. for a sample of observables



Observable	$\text{NLL}_{\ln\Sigma}$ discrepancy
$1-T$	$0.116^{+0.004}_{-0.004}\,\bar{\alpha}^3L^3$
vector p_t sum	$-0.349^{+0.003}_{-0.003}\,\bar{\alpha}^3L^3$
B_T	$-0.0167335\,\bar{\alpha}^2L^2$
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Occasionally the effect is postponed to NNLL at second order after azimuthal integration, and it shows up at NLL at third order

Conclusions

- A single shower must be accurate for different observables
  - necessary to develop a correspondence ingredients of the shower (branching probability, mapping, ordering), all-order amplitudes, and the logarithmic order
- We initiated such a study considering the family of dipole showers with local recoil
  - Asymptotic limits of the shower equations to establishing a connection to resummation
  - Differences in regions of phase space relevant for LL (subleading N_c) and NLL (leading N_c) in global, rIRC safe observables
- Ideally future developments should come with statements about how a given choice affect the all-order logarithmic structure

[Jadach, Kusina, Skrzypek, Slawinska '10; Nagy, Soper '12; Li, Skands '16; Hoeche, Krauss, Prestel '17; Hoeche, Prestel '17; Dulat, Hoeche, Prestel '18; Martinez, De Angelis, Forshaw, Plaetzer, Seymour '18]

  - Further developments necessary to test the accuracy of a shower at all orders
  - Establish a solid basis for the development of algorithms with higher accuracy
- Impact of tuning and pre-asymptotic effects important (perhaps dominant for some designs in phenomenological applications). Still a lot to understand in this direction

Thank you for listening

CAESAR: ordering variable

- The study of the logarithmic accuracy of parton showers requires a careful comparison with resummed calculations. The starting point is to build a resummation framework that is suitable for a MC formulation

- global and recursively IRC safe observables at NLL: CAESAR [Banfi, Salam, Zanderighi '01-'04]
  - resummation given by a shower of independent emissions off the Born legs strongly ordered in angle

e.g. e^+e^- -> p_1 p_2 + X

$$dP_n \simeq \frac{C_F^n}{n!}\prod_{i=1}^{n}\left(\frac{\alpha_s^{\mathrm{CMW}}(p_{\perp,i}^2)}{\pi}\frac{dp_{\perp,i}}{p_{\perp,i}}dz_i P_{q\to qg}(z_i)\frac{d\phi_i}{2\pi}\right)$$

Double-soft current integrated out inclusively

CAESAR: ordering variable

- The study of the logarithmic accuracy of parton showers requires a careful comparison with resummed calculations. The starting point is to build a resummation framework that is suitable for a MC formulation

- global and recursively IRC safe observables at NLL: CAESAR [Banfi, Salam, Zanderighi ’01-’04]
    - resummation given by a shower of independent emissions off the Born legs strongly ordered in angle

e.g. $e^+e^- \to p_1\, p_2 + X$

Use observable $v_i = V(k_i)$ as evolution variable
(not strictly necessary, it leads to a simpler structure)

$$dP_n \simeq \frac{C_F^n}{n!}\prod_{i=1}^{n}\left(\frac{\alpha_s^{\rm CMW}(p_{\perp,i}^2)}{\pi}\frac{dp_{\perp,i}}{p_{\perp,i}}dz_i P_{q\to qg}(z_i)\frac{d\phi_i}{2\pi}\right) \longrightarrow \Sigma(L) \equiv \int_0^{e^{-L}} dv'\,\frac{d\sigma}{dv'} \sim e^{-R(L)}\mathcal{F}_{\rm NLL}(\alpha_s L)$$

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- Sudakov radiator *R(v)* computed at NLL
- Effect of multiple emissions evaluated with LL (soft-collinear) matrix elements and observable

$$dP_n = \frac{C_F^n}{n!}\prod_{i=1}^{n}\left(\frac{2\alpha_s(p_{\perp,i}^2)}{\pi}\frac{dp_{\perp,i}}{p_{\perp,i}}\frac{dz_i}{1-z_i}\frac{d\phi_i}{2\pi}\right)$$

$$\mathcal{F}_{\rm NLL}(v) = \langle\Theta(1-\lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde{p}\},\{k_i\})}{v}))\rangle$$

Dipole showers: mapping

- The map is defined by (local recoil)

$$\tilde p_i + \tilde p_j \xrightarrow{\tilde p_i \to p_i + p_k} p_i + p_j + p_k$$

$$\begin{aligned} p_i^\mu &= \tilde z\, \tilde p_i^\mu + y\,(1-\tilde z)\,\tilde p_j^\mu + k_\perp \\ p_k^\mu &= (1-\tilde z)\,\tilde p_i^\mu + y\,\tilde z\,\tilde p_j^\mu - k_\perp^\mu \\ p_j^\mu &= (1-y)\,\tilde p_j^\mu \end{aligned}$$

see backup for branching probabilities

Pythia

- Evolution variable and branching:

$$v \equiv p_{\perp,\mathrm{evol}}$$

$$\rho^2_{\perp,\mathrm{evol}} = \frac{p^2_{\perp,\mathrm{evol}}}{(\tilde p_i + \tilde p_j)^2}\,, \qquad y = \frac{\rho^2_{\perp,\mathrm{evol}}}{z(1-z)}\,, \qquad \tilde z = \frac{(1-z)\left(z^2 - \rho^2_{\perp\mathrm{evol}}\right)}{z\,(1-z) - \rho^2_{\perp\mathrm{evol}}}$$

$$\rho_{\perp,\mathrm{evol}} \le z \le 1 - \rho_{\perp,\mathrm{evol}}$$

- k_t and rapidity of emission w.r.t. the emitter

$$\eta = \ln \frac{(1-\tilde z)Q}{|k_\perp|}\,, \qquad |k_\perp^2| = \frac{\left(z^2 - \rho^2_{\perp\mathrm{evol}}\right)\left((1-z)^2 - \rho^2_{\perp\mathrm{evol}}\right)}{\left(z\,(1-z) - \rho^2_{\perp\mathrm{evol}}\right)^2}$$

Dire

- Evolution variable and branching:

$$v \equiv \sqrt{t}$$

$$\kappa^2 = \frac{t}{(\tilde p_i + \tilde p_j)^2}\,, \qquad y = \frac{\kappa^2}{1-z}\,, \qquad \tilde z = \frac{z-y}{1-y}$$

$$\frac{1}{2} - \sqrt{\frac{1}{4} - \kappa^2} \le z \le \frac{1}{2} + \sqrt{\frac{1}{4} - \kappa^2}$$

- k_t and rapidity of emission w.r.t. the emitter

$$\eta = \ln \frac{(1-\tilde z)Q}{|k_\perp|}\,, \qquad |k_\perp^2| = (1-z)\,\frac{z(1-z) - \kappa^2}{(1-z-\kappa^2)^2}\,t$$

Dipole showers: branchings

- We focus on k_t-ordered dipole showers with local recoil (most common design today)
  - Consider the designs of `Pythia8`'s shower and `Dire`. The map is defined by

$$\tilde{p}_i + \tilde{p}_j \xrightarrow{\tilde{p}_i \to p_i + p_k} p_i + p_j + p_k$$

$$\begin{aligned} p_i^\mu &= \tilde{z}\,\tilde{p}_i^\mu + y\,(1-\tilde{z})\,\tilde{p}_j^\mu + k_\perp \\ p_k^\mu &= (1-\tilde{z})\,\tilde{p}_i^\mu + y\,\tilde{z}\,\tilde{p}_j^\mu - k_\perp^\mu \\ p_j^\mu &= (1-y)\,\tilde{p}_j^\mu \end{aligned}$$

`Pythia`

$$d\mathcal{P}_{q\to qg} = \frac{\alpha_s(p^2_{\perp,\mathrm{evol}})}{2\pi}\,\frac{dp^2_{\perp\mathrm{evol}}}{p^2_{\perp\mathrm{evol}}}\,dz\,\frac{d\phi}{2\pi}\,C_F\left(\frac{1+z^2}{1-z}\right)$$

$$d\mathcal{P}_{g\to gg} = \frac{\alpha_s(p^2_{\perp,\mathrm{evol}})}{2\pi}\,\frac{dp^2_{\perp\mathrm{evol}}}{p^2_{\perp\mathrm{evol}}}\,dz\,\frac{d\phi}{2\pi}\,\frac{C_A}{2}\left[\frac{1+z^3}{1-z}\right]$$

$$d\mathcal{P}_{g\to q\bar{q}} = \frac{\alpha_s(p^2_{\perp,\mathrm{evol}})}{2\pi}\,\frac{dp^2_{\perp\mathrm{evol}}}{p^2_{\perp\mathrm{evol}}}\,dz\,\frac{d\phi}{2\pi}\,\frac{n_f T_R}{2}\,\mathcal{D}\,[1-2\tilde{z}\,(1-\tilde{z})]$$

$$\mathcal{D} = (1-x)^2(1+x)\,, \qquad x \equiv \frac{(p_i+p_k)^2}{(\tilde{p}_i+\tilde{p}_j)^2}$$

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`Dire`

$$\begin{aligned} d\mathcal{P}_{q\to qg} &= \frac{\alpha_s(t)}{2\pi}\frac{dt}{t}\,dz\,\frac{d\phi}{2\pi}\,C_F\,\left[2\frac{1-z}{(1-z)^2+\kappa^2}-(1+z)\right] \\ d\mathcal{P}_{g\to gg} &= \frac{\alpha_s(t)}{2\pi}\frac{dt}{t}\,dz\,\frac{d\phi}{2\pi}\,\frac{C_A}{2}\,\left[2\frac{1-z}{(1-z)^2+\kappa^2}-2+z(1-z)\right] \\ d\mathcal{P}_{g\to q\bar{q}} &= \frac{\alpha_s(t)}{2\pi}\frac{dt}{t}\,dz\,\frac{d\phi}{2\pi}\,\frac{n_f T_R}{2}\,[1-2z(1-z)] \end{aligned}$$

Difference between shower and NLL

$$\delta\Sigma^{(2\,\mathrm{emissions})}(L) = \left(C_F\frac{2\alpha_s}{\pi}\right)^2 \int_0^1 \frac{dv_1}{v_1}\int_{\ln v_1}^{\ln 1/v_1} d\eta_1 \int_0^{v_1}\frac{dv_2}{v_2}\int_{\ln v_2}^{\ln 1/v_2} d\eta_2 \int_0^{2\pi}\frac{d\phi_1}{2\pi}\int_0^{2\pi}\frac{d\phi_2}{2\pi}\times$$
$$\times\left[\Theta\big(e^{-L} - V(p_1^{\mathrm{shower}}, p_2)\big) - \Theta\left(e^{-L} - V(p_1^{\mathrm{correct}}, p_2)\right)\right]$$

$$\delta\Sigma^{(3\,\mathrm{emissions})}(L) = \left(C_F\frac{2\alpha_s}{\pi}\right)^3 \int_0^1 \frac{dv_1}{v_1}\int_0^{v_1}\frac{dv_2}{v_2}\int_0^{v_2}\frac{dv_3}{v_3}\int_{\ln v_1}^{\ln 1/v_1} d\eta_1 \int_{\ln v_2}^{\ln 1/v_2} d\eta_2 \int_{\ln v_3}^{\ln 1/v_3} d\eta_3\times$$
$$\times\int_0^{2\pi}\frac{d\phi_1}{2\pi}\int_0^{2\pi}\frac{d\phi_2}{2\pi}\int_0^{2\pi}\frac{d\phi_3}{2\pi}\times$$
$$\times\Big[\Theta(e^{-L} - V(p_1^{\mathrm{shower}}, p_2^{\mathrm{shower}}, p_3)) - \Theta(e^{-L} - V(p_1^{\mathrm{correct}}, p_2^{\mathrm{correct}}, p_3))$$
$$-\Theta(e^{-L} - V(p_1^{\mathrm{shower}}, p_2)) + \Theta(e^{-L} - V(p_1^{\mathrm{correct}}, p_2))$$
$$-\Theta(e^{-L} - V(p_1^{\mathrm{shower}}, p_3)) + \Theta(e^{-L} - V(p_1^{\mathrm{correct}}, p_3))$$
$$-\Theta\big(e^{-L} - V(p_2^{\mathrm{shower}}, p_3)\big) + \Theta\left(e^{-L} - V(p_2^{\mathrm{correct}}, p_3)\right)\Big]$$