# Measurement of Entanglement Spectrum on IBM Quantum Computer

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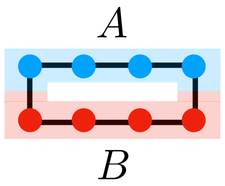
## Introduction

**Definition of Entanglement Spectrum**
Eigenvalues of the reduced density matrix of a subsystem, i.e.

$\rho$  $\rho_A = \mathrm{tr}_B \rho$



**Application: Useful Tool in identifying Symmetry Protected Topological (SPT):** The bipartitioning used to define the reduced density matrix introduces a virtual bulk-boundary correspondence leading to protected degeneracies in the entanglement spectrum.

**Problem:** Obtaining the entanglement spectrum is experimentally challenging. Only recently with the improvements in trapped ions and superconducting quantum simulators can the entanglement entropy[2] or density matrix be accessed[3].

**Solution:** Quantum Computers allows one to efficiently prepare grounds states of local hamiltonians. The ability to individually measure qubits then provides access to the reduced density matrix and hence the entanglement spectra.
**We attempt to use the IBM quantum cloud computing resource to perform such a task.**

[1]N. Friis et. al. Phys. Rev. X 8, 021012 (2018)
[2]R. Islam et. al. Nature 528, 77 EP (2015).

## Models

**1) Trivial Paramagnet:** This is simply the state which is polarised in the x-direction.

$$|\mathrm{PM}\rangle = \frac{1}{\sqrt{2^N}} \sum_{\boldsymbol{r}} |r\rangle = |+\rangle^{\otimes N}$$

**2) Cat State:** aka "GHZ" state

$$|\mathrm{Cat}\rangle = \frac{1}{\sqrt{2}}(|00\cdots\rangle + |11\cdots\rangle)$$

**3) Topological Paramagnet (SPT):** This is given by the ground state of the following stabiliser Hamiltonian

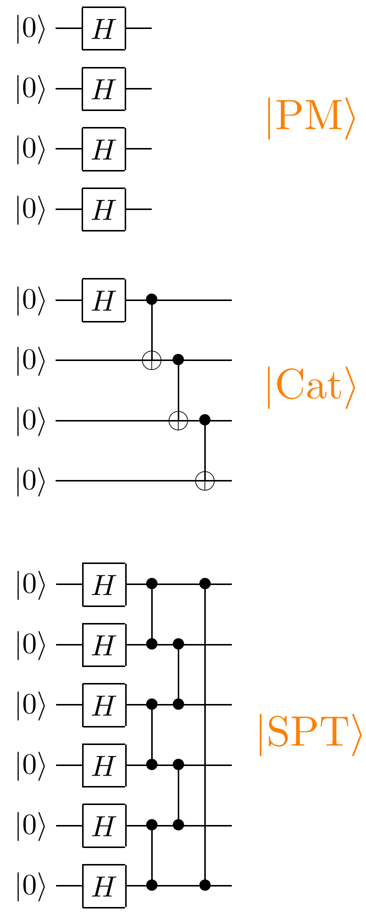
$$H_{\mathrm{SPT}} = -\sum_i \sigma_{z,i-1}\sigma_{x,i}\sigma_{z,i+1}$$

such that

$$\sigma_{z,i-1}\sigma_{x,i}\sigma_{z,i+1}|SPT\rangle = |SPT\rangle$$

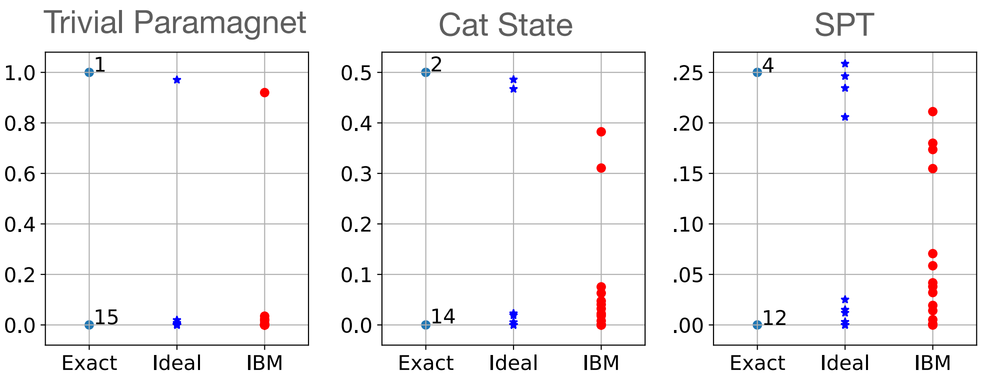
When defined on a chain with open boundary conditions, there would be a topological fourfold degeneracy. For periodic boundary conditions, the ground state is unique but a fourfold degeneracy is still present in the entanglement spectrum

### Quantum Circuits



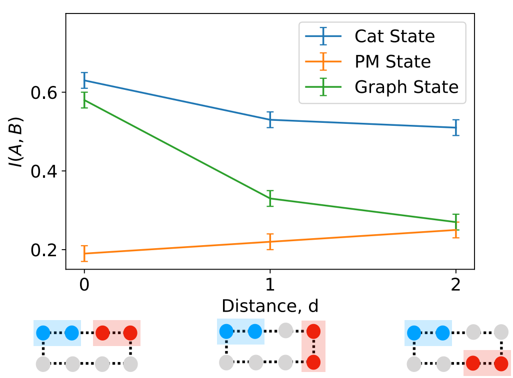

## Results

**Using the IBM 16 qubit quantum computer (ibmqx5):** We use 8 of the 16 bits to construct the states corresponding to the three different models. After which, we performed full state tomography of 4 of the 8 bits so as to construct the reduced density matrix. The eigenvalues of the corresponding reduced density matrices are shown in the 3 figures below.




Here the four fold degeneracy of the SPT is relatively apparent.

**Mutual information between separated subsystems:** Here we can see that the Cat state is long ranged entangled whereas the Graph(SPT) state is only short range entangled.




## Conclusion and Outlook

- **There is hope that Quantum Computers could (not yet) be a useful tool for quantum simulations.** Google has recently announced its 72 bit quantum processor with better fidelities than the IBM device. With such rapid developments, the hope for useful quantum simulations on a quantum device might not be too far in the future.
- **The entanglement spectra of a small subsystem can already be a useful diagnostic for quantum phases.** In practice is only necessary for the subsystem size to exceed the characteristic correlation length of the system. For gapped Hamiltonians, such lengths scales would be independent of system size.
- **The protocol used here is clearly not feasible for obtaining entanglement spectra for large subsystems however,** there is a more efficient algorithm[1] capable of obtaining the low lying part of the spectra with polynomial resources. But there is some overhead in implementing the protocol so this was not attempted.
- **Even though quantum devices are not useful for quantum simulations yet, there are other applications such as quantum machine learning which could possibly make use of a noisy quantum device without error correction[2].**

[1]S. Johri et. al. Phys. Rev. B 96, 195136 (2017)
[2]W. Huggins et. al. arXiv:1803.11537 (2018)

see: Phys. Rev. Lett. **121**, 086808 (2018)