PHY 117 HS2023

Week 3, Lecture 2
Oct. 4th, 2023
Prof. Ben Kilminster

week 2 quiz

Question

What is the direction of the angular momentum vector for the second hand of an SBB clock ?

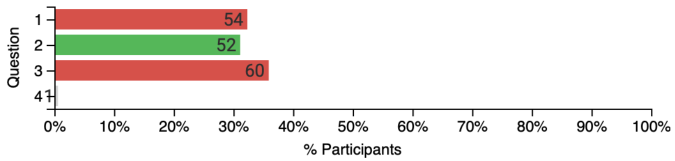
∅

- into the clock
- out of the clock
- around, in a circle

Key figures

Participants	167
Max Score	1.0
Number of correct answers	52
Number of wrong answers	115
Item difficulty	0.31
Average score	0.31
Average completion time	2 min 29 sec

Key figures

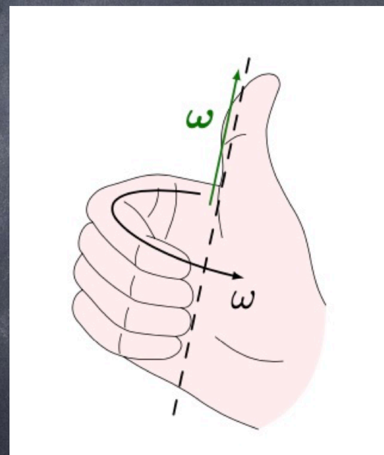


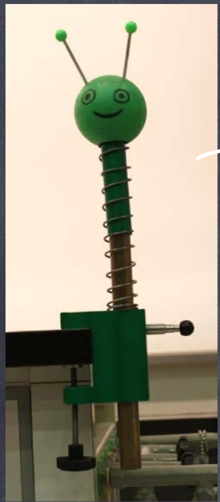
× 1. (0 points)
around, in a circle

✓ 2. (1 points)
into the clock

× 3. (0 points)
out of the clock

× 4. Participant doesn't give an answer





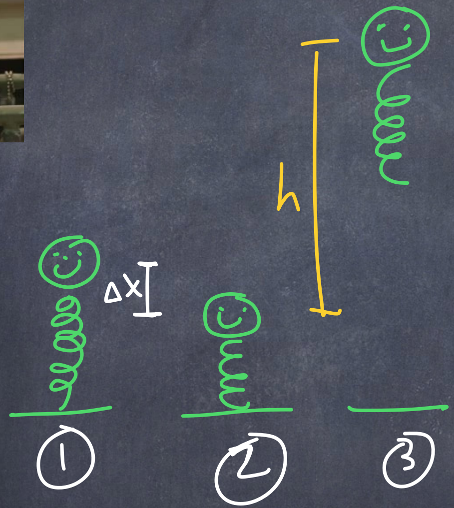
Yesterday

How high does the grasshopper jump?

$F_s = -k\Delta x$

force points opposite the stretching of the spring.

$$k = \frac{F_s}{\Delta x} = \frac{(2.5\,kg)(9.81\,m/s^2)}{0.04\,m} = 612.5\,\frac{N}{m}$$



$$-\Delta U = W = \int \bar{F}\cdot d\bar{s}$$

$$W = \int_0^x F\cdot dx = \int_0^x (-kx)\cdot dx$$

$$W = -\frac{1}{2}kx^2$$

$$\Delta U = -W = \frac{1}{2}kx^2$$

$$\boxed{U = \frac{1}{2}kx^2}$$

energy stored in a spring

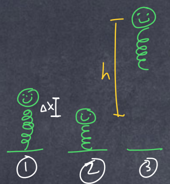
where x is the compression of spring

There are 3 types of energy here:

U_g : gravitational potential energy, $U_g = mgh$

U_s : spring potential energy, $U_s = \frac{1}{2}k(\Delta x)^2$

K : kinetic energy, $K = \frac{1}{2}mv^2$



At ②, all energy is U_s. Between ② + ③, energy is a combination of U_s, U_g, K. And at ③, it is all U_g.

Applying energy conservation:

$$E_{②} = E_{③}$$

$$U_s = U_g$$

$$\frac{1}{2}k(\Delta x)^2 = mgh \implies h = \frac{\frac{1}{2}k(\Delta x)^2}{mg}$$

we know:

$k = 612 \frac{N}{m}$

m = mass of grasshopper
$= 0.424$ kg

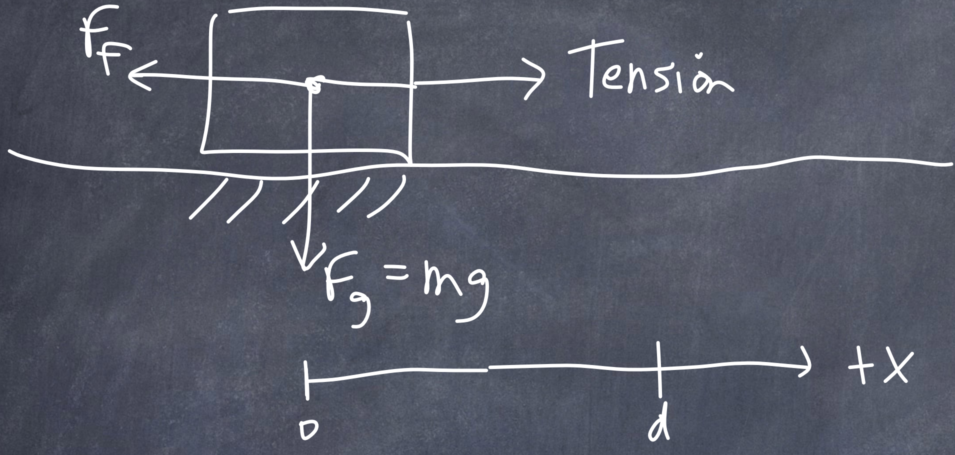
Δx = compression of spring
$= 0.14$ m

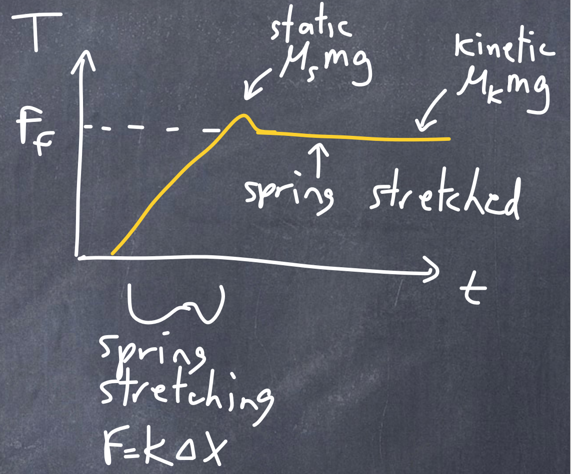
$$h = \frac{\frac{1}{2}\left(612 \frac{N}{m}\right)(0.14\ m)^2}{(0.424\ kg)(9.8\ m/s^2)} = \boxed{1.44\ m \text{ prediction}}$$

h = measurement $= 1.5$ m

work can be done by friction:

F_f is opposite movement
(−)





$$F_f = -\mu F_N = -\mu mg$$

$$W = \int_0^d \bar{F}\cdot d\bar{x} = \int_0^d F_f\,dx = \int_0^d -\mu mg\,dx$$

$$W = -\mu mgx\Big]_0^d =$$

$$\boxed{W = -\mu mgd}$$

work done by the friction for the case of moving a distance d

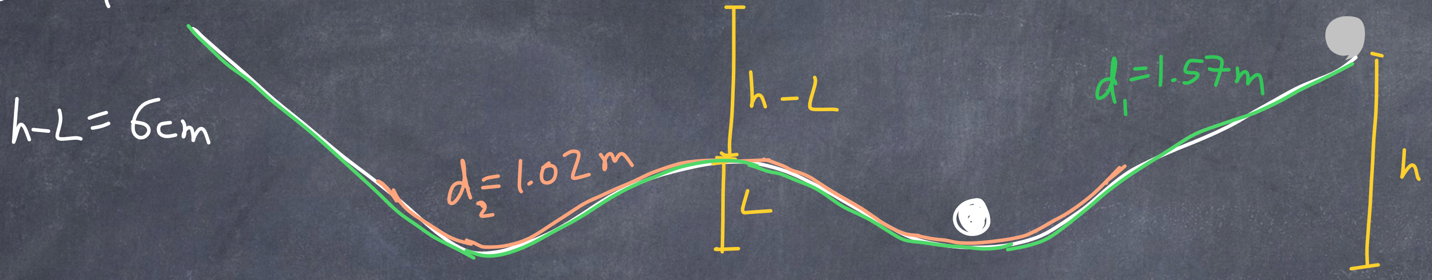
At constant velocity,

$$\sum F = 0$$

$$T - F_f = 0$$

$T = F_f$ is constant

How many times will a steel ball roll across this track before getting stuck?



This problem can be solved with energy conservation we have K, U_g, and the work due to friction, W_f. The ball will roll across the track until it loses gravitational energy through friction, and can't get over the bump. This happens when $U_g = mg(h-L)$ is lost due to the work of friction, $W_f = \mu mgD$

D = total distance traveled

μ = coefficient of friction for a steel ball rolling on an aluminum track $= \mu = 0.00076$.

d_1, d_2 are lengths of the track at the start and the end.

End is when $W_f = U_g$

$$\mu \cancel{m}\cancel{g} D = \cancel{m}\cancel{g}(h-L)$$

$$D = \text{total distance} = \frac{h-L}{\mu}$$

so $D = \dfrac{0.06\ m}{0.00076} = 79\ m$

How many times across is 79 m?

Length of the track will vary from 1.57 to 1.02 m,

so the average length is $\langle d \rangle = \dfrac{1.57 + 1.02\ m}{2} = 1.3\ m$

so $\dfrac{D}{\langle d \rangle} = \#\ \text{times} = 61\ \text{times}$

prediction

measured = ? = 56

Collisions

Elastic and inelastic collisions

A collision is elastic if there is no work done by the forces of friction, deformation, or sticking or breaking. Otherwise, it is inelastic.

In elastic collisions, $K + U = \text{constant}$

If U is the same before and after, then K must also be the same. $K_{initial} = K_{final}$

So Kinetic energy is conserved in elastic collisions

Elastic collisions : $K_{initial} = K_{final}$

In elastic collisions, momentum is also conserved.

Momentum is defined as $\bar{p} = m\bar{v}$ (Its a vector)

Conservation of momentum means

If same objects, then $m_i = m_f$

$$\bar{p}_{initial} = \bar{p}_{final} \quad \text{or} \quad \underbrace{\sum m_i \bar{v}_i}_{\text{total initial momentum}} = \underbrace{\sum m_f \bar{v}_f}_{\text{total final momentum}}$$

__Experiment:__

initial

$\boxed{m_1} \rightarrow v_1$ $\boxed{m_2}$ $v_2 = 0$

$m_1 = m_2$

Final ?

$\boxed{m_1}$ $v_1^f = ?$ $\boxed{m_2}$ $v_2^f = ?$

conservation of momentum: $m_1\vec{v}_1 = m_1\vec{v}_1^f + m_2\vec{v}_2^F$ (1)

conservation of kinetic energy: $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v_1^{f2} + \frac{1}{2}m_2v_2^{f2}$ (2)

Rewrite (1): $m_1(v_1 - v_1^f) = m_2v_2^f$ (3)

Rewrite (2): $m_1(v_1^2 - v_1^{f2}) = m_2v_2^{f2}$

$m_1(v_1 - v_1^f)(v_1 + v_1^f) = m_2v_2^{f2}$ (4)

$m_1v_1^2 - m_1v_1^{f2} = m_2v_2^{f2}$

$m_1(v_1^2 - v_1^{f2}) = m_2v_2^{f2}$

factorize: $a^2 - b^2 = (a+b)(a-b)$

Divide (4) by (3): $v_1 + v_1^f = v_2^f$ (5)

Substitute (5) → (3): $m_1(v_1 - v_1^f) = m_2(v_1 + v_1^f)$

$m_1v_1 - m_1v_1^f = m_2v_1 + m_2v_1^f$

$v_1(m_1 - m_2) = v_1^f(m_1 + m_2)$

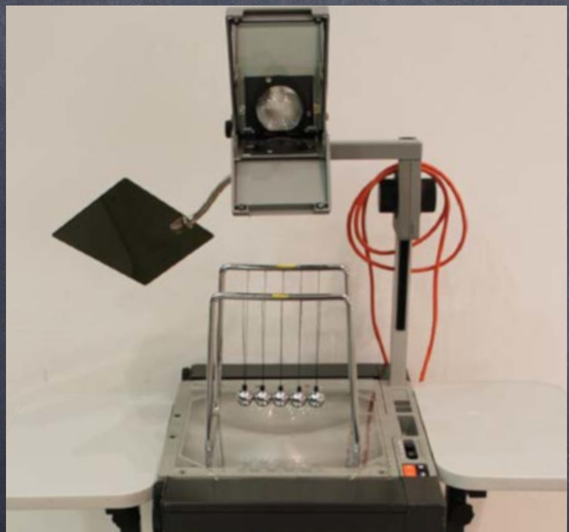
$v_1^f = v_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)$ (6)

In our case $m_1 = m_2$, so (6) becomes $v_1^f = 0$

If $v_1^f = 0$, then by (1) → $v_2^f = v_1$
(and $m_1 = m_2$)

So car 1 will stop, and car 2 will continue with same velocity as car 1 initial

kinetic
momentum and energy (approximately)
conserved

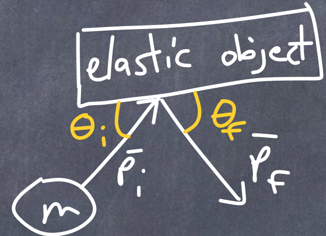


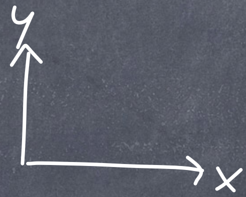
Approximately elastic

What if the collision is inelastic ?

K is not conserved.

However, $\bar{p}$ is conserved.



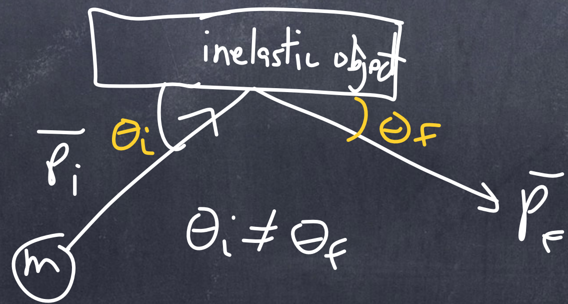


Elastic collision : $\theta_i = \theta_f$

x: $p_{i_x} = p_{i_f}$

y: $p_{i_y} = -p_{f_y}$

Q: why does object change directions, $\Delta p \neq 0$? (see Impulse...)



$\theta_i \neq \theta_f$

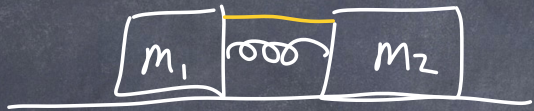
$|\bar{p}_i| \neq |\bar{p}_f|$

what happens?

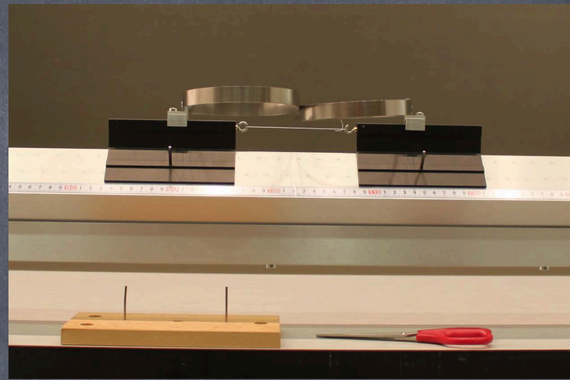
we need to account for the deformation or movement of inelastic object.

Other momentum conservation examples:

①



initial state
$\Sigma \bar{p} = 0$



final momentum $= 0$

$$0 = m_1 v_1 + m_2 v_2$$

$$m_1 v_1 = -m_2 v_2$$

If $m_1 = m_2$, then $v_1 = -v_2$

②

$M > m_1$

$$p_i = 0 = p_F$$

final: $p_F = 0 = m_1 v_1 + MV$

$$v_1 = -V\left(\frac{M}{m_1}\right)$$

Relation of momentum and force:

$$\bar{p} = m\bar{v}$$

$$\frac{d\bar{p}}{dt} = m\frac{d\bar{v}}{dt} \quad \text{(If mass doesn't change)}$$

$\uparrow$ but this is $\bar{a} = \dfrac{d\bar{v}}{dt}$

so $\dfrac{d\bar{p}}{dt} = m\bar{a} = \bar{F} \Rightarrow$ A net force will change an object's momentum.

Momentum conservation:

So $\bar{F} = \dfrac{d\bar{p}}{dt} \rightarrow d\bar{p} = \bar{F}\,dt \Rightarrow \Delta\bar{p} = \displaystyle\int_{t_1}^{t_2} F\,dt$

so where $\Delta\bar{p}$ = change in momentum $= \bar{p}_2 - \bar{p}_1$

If $\Delta p = 0 \Rightarrow$ momentum is conserved if <u>no</u> net force

Let's look at forces changing with time.

$$\bar{F} = \frac{d\bar{p}}{dt} \qquad d\bar{p} = \bar{F}\,dt$$

If the force is constant, then $\Delta p = F \Delta t$

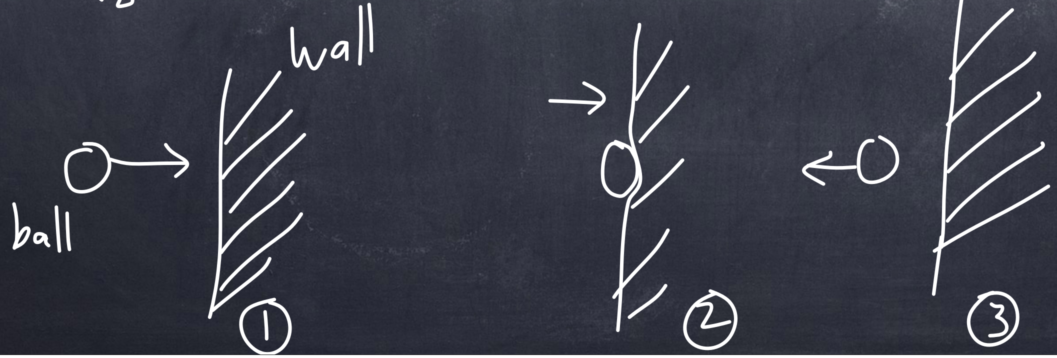
constant force

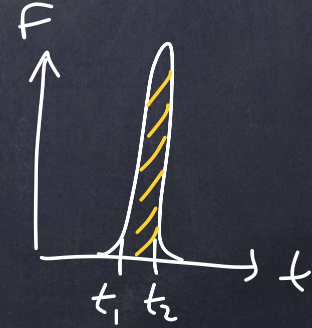
amount of time of force.

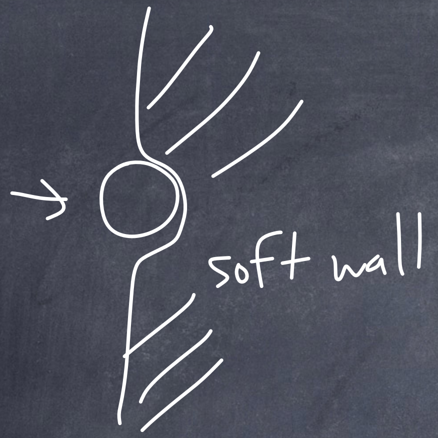
Change in momentum is the area.

If the force is not constant, then

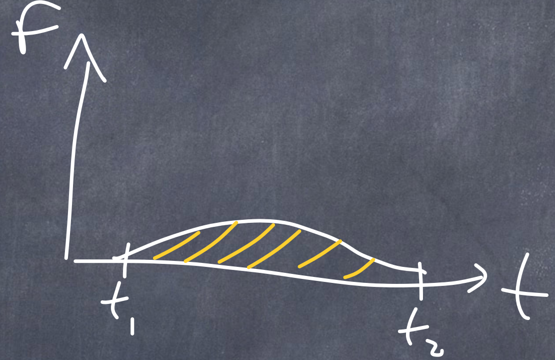
then $dp = \int_{t_1}^{t_2} F(t)\,dt$ = area under the curve of F vs. t from t_1 to t_2





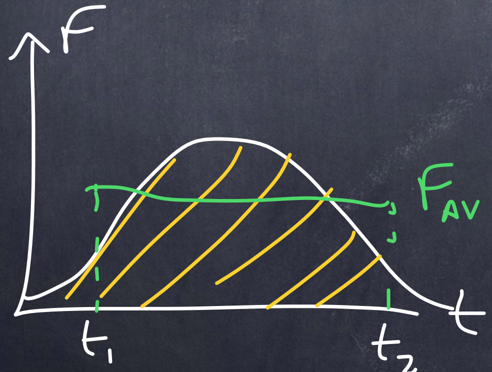


In this case, Δt is longer, so the contact is longer



The force is more constant with time.

$$\Delta p = \int F(t)\,dt = \text{Impulse} = I$$



Time average of force $= F_{AV}$ is equivalent to a constant force over time.

$$(F_{AV})(\Delta t) = \int_{t_1}^{t_2} F(t)\,dt$$

we can minimize the average force in a collision if Δt is long or if the objects deform or break. (Inelastic collision)

Questions after class:

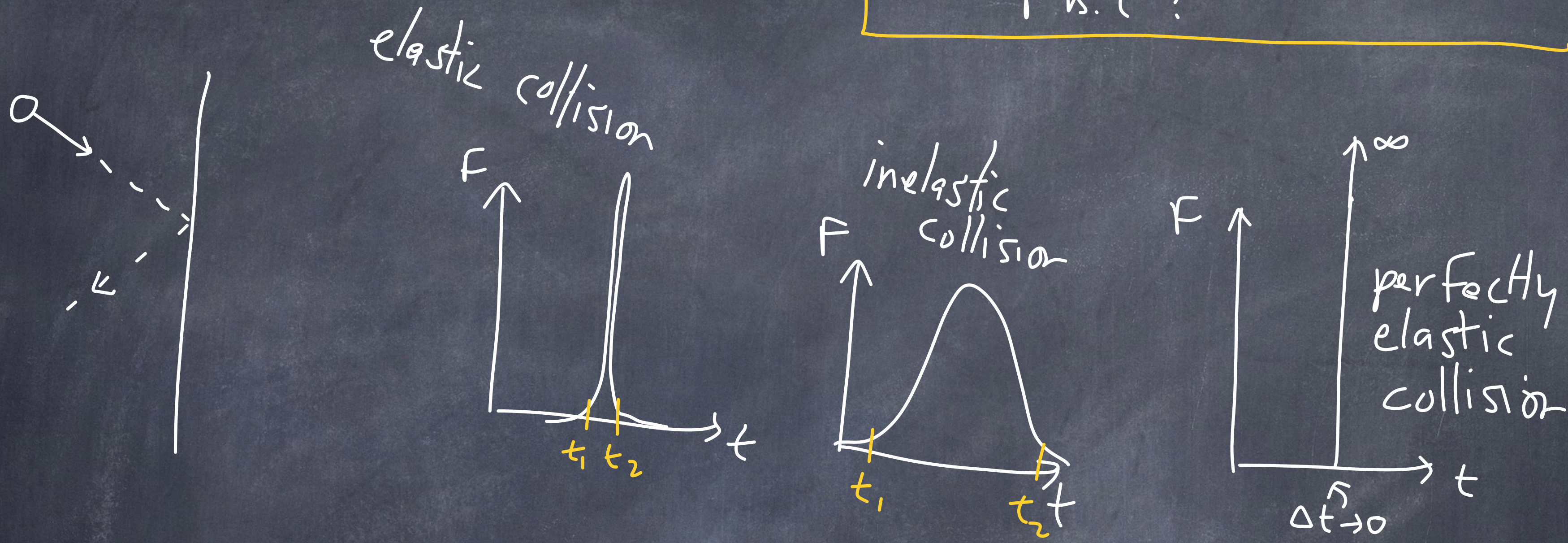
1) what is the difference between elastic & inelastic collisions in terms of F vs. t ?

2) If momentum changes directions, isn't the force always the same ?

3) How can something change directions? Isn't momentum <u>not</u> conserved if this happens?

4) what if the problem is 2-dimensional? How is momentum conserved?

1) what is the difference between elastic & inelastic collisions in terms of F vs. t ?



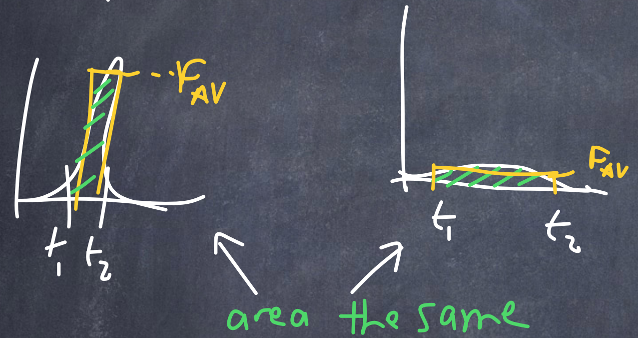
For perfectly elastic collisions,
$\Delta t \to 0$, $F \to \infty$
(these don't happen in the real world.
All interactions have some Δt)

$$\Delta p = I = F_{AV}\,\Delta t$$

2) If momentum changes directions, isn't the force always the same?

The average force depends on Δt.

either

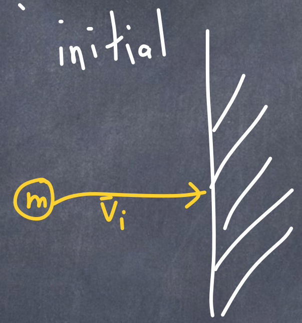


Notice, we can have the same $\Delta p = p_2 - p_1$ in cases where Δt is small and F_{AV} is large, or when Δt is large and F_{AV} is small.

In both cases Δp is the same (area is the same)

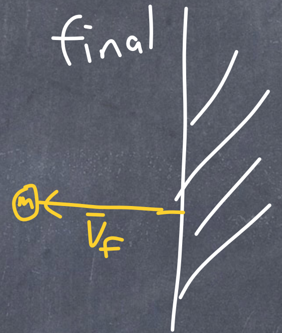
How can something change directions? Isn't momentum not conserved if this happens?

Consider this case:



$\bar{v}_i = -\bar{v}_f$

$|\bar{v}_i| = |\bar{v}_f|$



The momentum changes from mv_i to mv_f

$\bar{p}_f = -\bar{p}_i$ (1) substitute in (1)

What happened?

$$\Delta \bar{p} = \bar{p}_2 - \bar{p}_1 = p_f - p_i = -\bar{p}_i - \bar{p}_i = -2\bar{p}_i$$

The momentum changed.

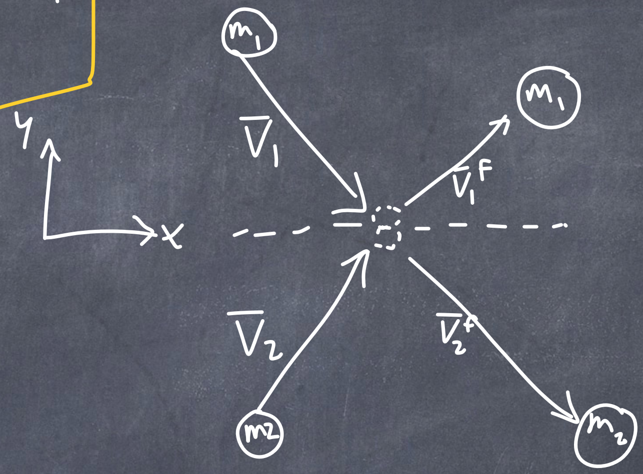
How? The wall provides a force F_{AV} for a time Δt

$$\Delta p = F_{AV}\Delta t \Longrightarrow \bar{F}_{AV} = \frac{-2\bar{p}_i}{\Delta t}$$

If we know Δt, we can calculate F_{AV}

④ What if the problem is 2-dimensional? How is momentum conserved?

consider this inelastic collision



Momentum will be conserved in all directions:

Therefore, $\Sigma \bar{p}_i = \Sigma \bar{p}_f$ we have these equations.

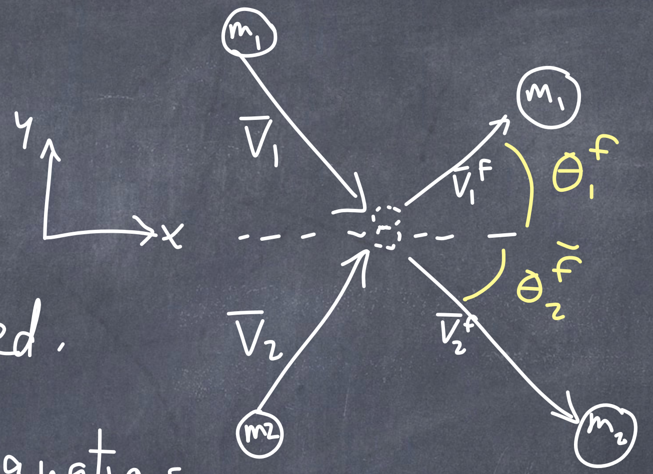
$$\Sigma p_{x_i} = \Sigma p_{x_f} \longrightarrow \begin{cases} m_1 v_{1_x} + m_2 v_{2_x} = m_1 v_{1_x}^f + m_2 v_{2_x}^f \\ m_1 v_{1_y} + m_2 v_{2_y} = m_1 v_{1_y}^f + m_2 v_{2_y}^f \end{cases}$$

$$\Sigma p_{y_i} = \Sigma p_{y_f}$$

IF we know the masses and initial velocities, we would have 4 unknowns and 2 equations. So we need more information to solve this.

④ Continued...

If the collision were also elastic, then K would be conserved.



Then we would have 3 equations and 4 unknowns:

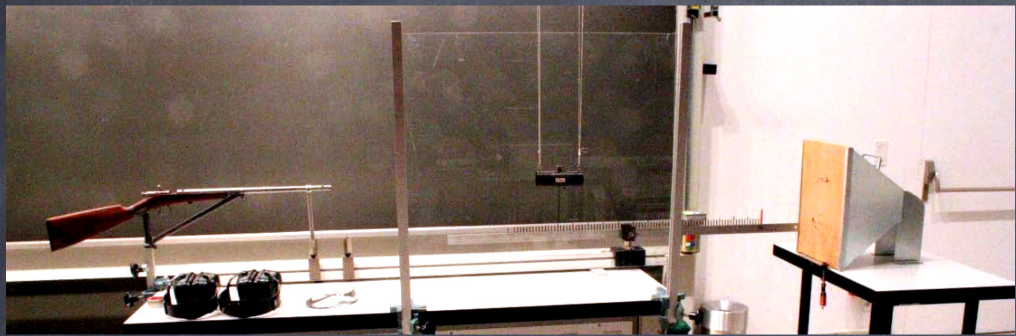
Kinetic energy conservation: $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1^{f^2} + \frac{1}{2}m_2v_2^{f^2}$

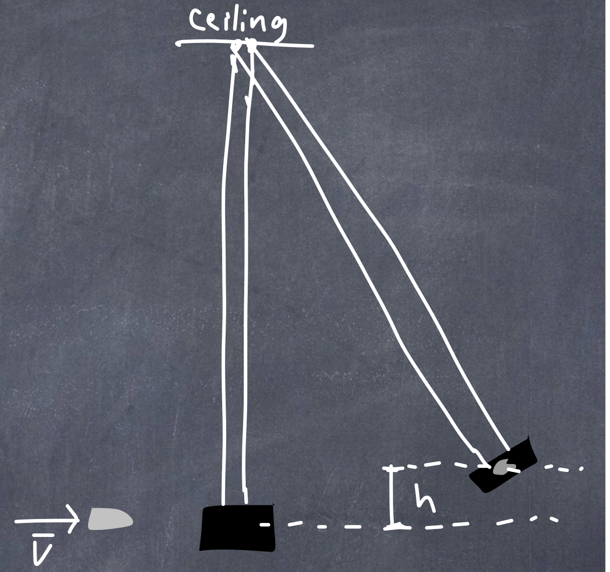
momentum conservation in x: $m_1V_{1_x} + m_2V_{2_x} = m_1V_{1_x}^f + m_2V_{2x}^f$

momentum conservation in y: $m_1V_{1_y} + m_2V_{2_y} = m_1V_{1_y}^f + m_2V_{2_y}^f$

Knowing one of the angles, θ_1^f or θ_2^f, or one final velocity component $(V_{1x}^f, V_{2x}^f, V_{1y}^F, V_{2y}^f)$ would be enough information to solve the problem.

Ballistic pendulum experiment

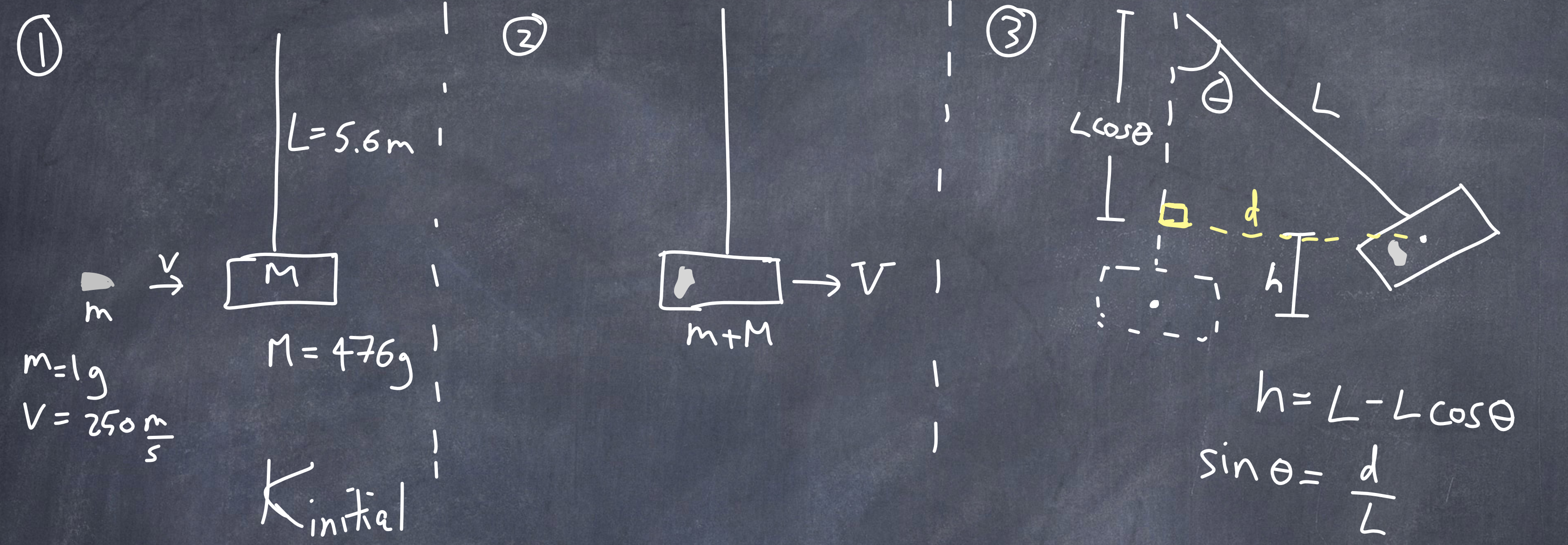




How high will the block go?

video posted soon!

Ballistic Pendulum – Momentum conservation in inelastic collision.



Between ① + ②, momentum is conserved. **Inelastic collision**

Kinetic energy is not conserved.

$$mv = (m+M)V$$

V: velocity of block + bullet.

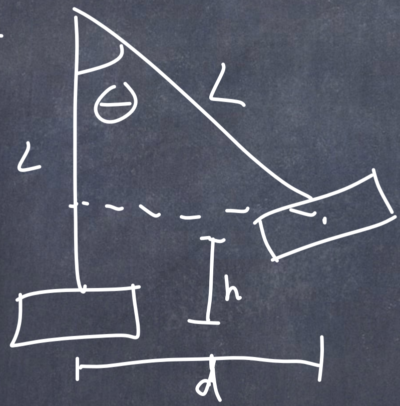
Between ② + ③, this is elastic.

$$K_{②} = U_{③}$$

$$\underbrace{\frac{1}{2}(\cancel{m+M})V^2}_{K_2} = \underbrace{(\cancel{m+M})gh}_{U_3}$$

solve for $h = \dfrac{\frac{1}{2}V^2}{g} = \dfrac{\frac{1}{2}\left[\frac{m}{m+M}v\right]^2}{g} = \boxed{\text{predicted } h \quad 0.014\,m}$

$L = 5.6\,m$



we measure d.

$h = L - L\cos\theta$

$\sin\theta = \dfrac{d}{L} \Rightarrow \theta = \sin^{-1}\left(\dfrac{d}{L}\right)$

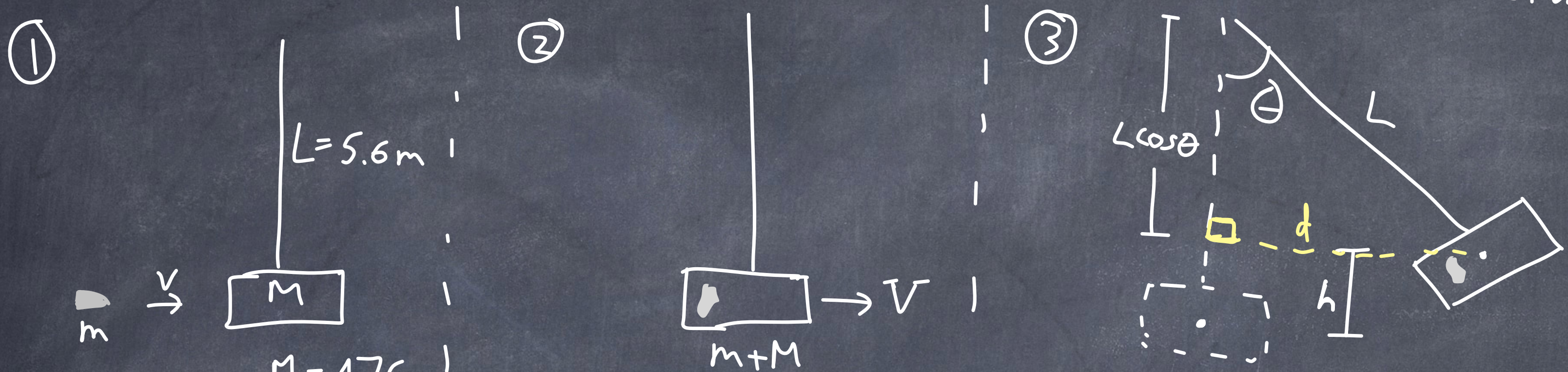
we can measure d, figure out θ, then figure out h.

measure $d = 0.4\,m$, $\theta = \sin^{-1}\left(\dfrac{0.4m}{5.6m}\right) = 0.07$ radians

and calculate $h = L - L\cos(0.07\text{ rad}) = 5.6\,m - 5.585$

$\boxed{\text{measured } h = 0.014\,m}$

Ballistic Pendulum - Momentum conservation in inelastic collision.



$m = 1g$

$V = 250 \frac{m}{s}$

$M = 476g$

$K_{initial}$

$h = L - L\cos\theta$

$\sin\theta = \frac{d}{L}$

Note:

If kinetic energy was conserved, then

$K_{①} = U_{③}$

$\frac{1}{2}mv^2 = mgh \longrightarrow h = \frac{v^2}{2g}$

$\underline{\underline{h = 6.7\,m}}$ wrong!

(collision is inelastic)