# Exact many-body quantum scar states with topological properties in dimensions 1, 2, and 3


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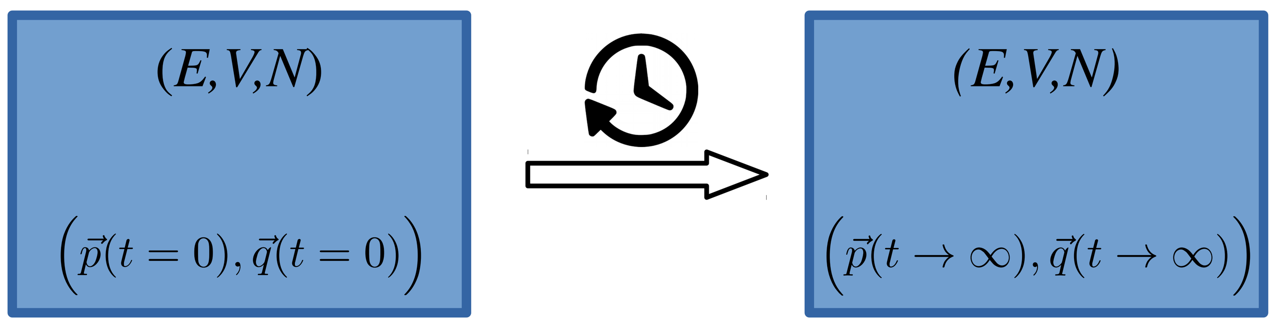
## 1. Background: Fundamental assumptions in thermodynamics

- Classical thermodynamics relies on the assumption of **ergodicity**.
- The corresponding assumption of ergodicity in quantum thermodynamics is **eigenstate thermalization hypothesis**.
- In thermodynamics, we compute the average of a quantity over detailed states, instead of the value relaxed over infinite amount of time under the detailed microscopic mechanics.

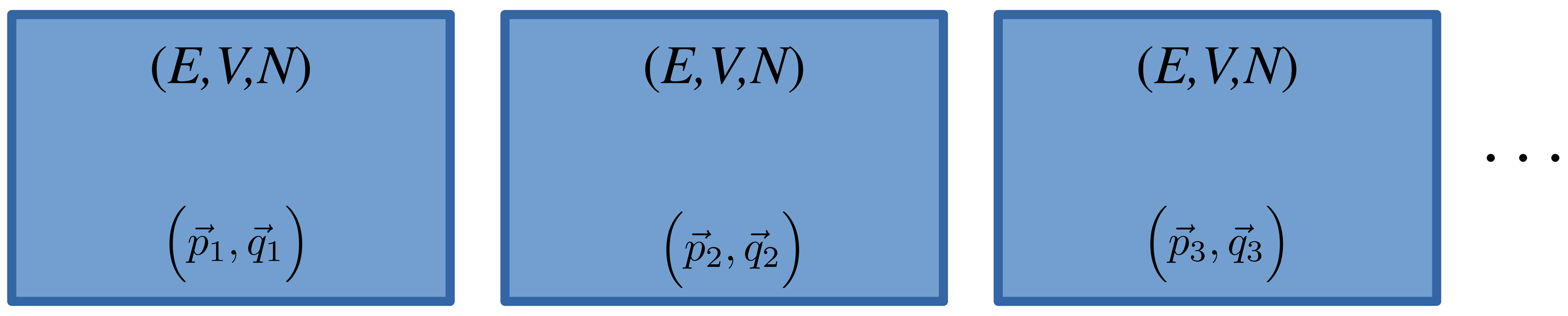
## 2-1. Ergodicity

$$H(\vec{p},\vec{q}) \in (E-\Delta E, E+\Delta E)$$




Average of a quantity over infinite time

$$\bar{A} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \mathrm{d}t \; A(\vec{p}(t), \vec{q}(t))$$



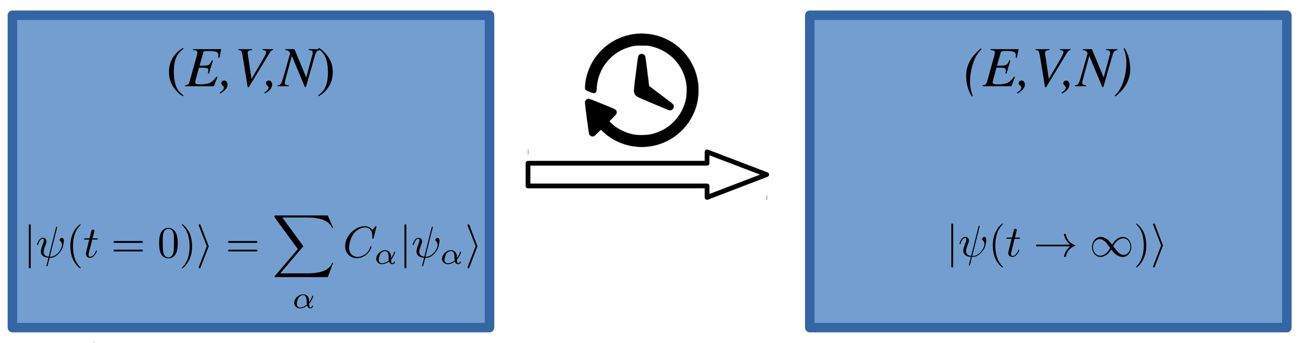

Average of a quantity over detailed states

$$\langle A \rangle = \frac{1}{\Gamma(E,V,N)} \int_\Gamma \mathrm{d}^{3N}p \; \mathrm{d}^{3N}q \; A(\vec{p},\vec{q})\, \rho_{\mathrm{mc}}(\vec{p},\vec{q})$$

Ergodicity assumes $\bar{A} = \langle A \rangle$

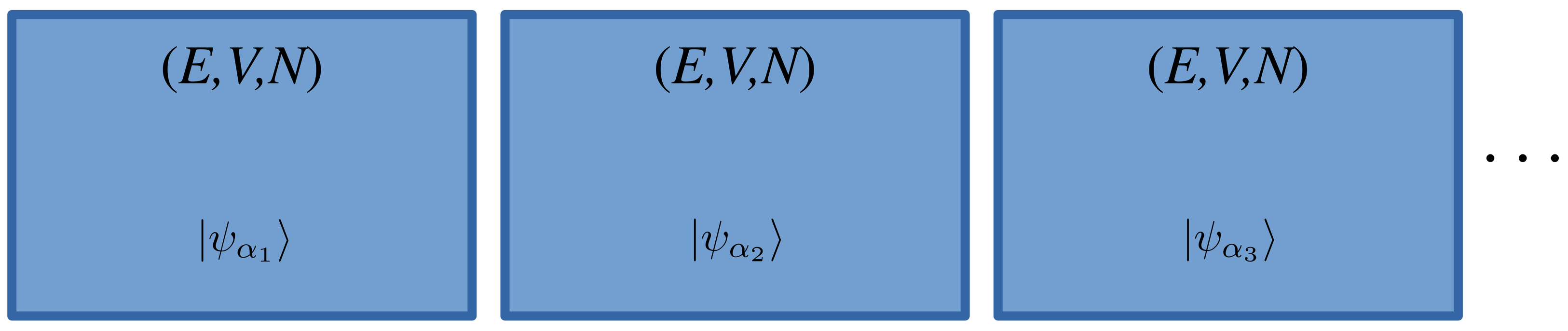
## 2-2. Eigenstate thermalization hypothesis (ETH)

$$\langle \psi_\alpha | \hat{H} | \psi_\alpha \rangle \in (E-\Delta E, E+\Delta E)$$




Average of an observable expectation value over infinite time

$$\bar{A}_{\mathrm{QM}} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \mathrm{d}t \; \langle \psi(t) | \hat{A} | \psi(t) \rangle = \ldots = \sum_\alpha |C_\alpha|^2 A_{\alpha\alpha}$$




Average of an abservable expectation value over detailed states

$$\langle A \rangle_{\mathrm{QM}} = \sum_\alpha \langle \psi_\alpha | \hat{A} | \psi_\alpha \rangle = \ldots = \frac{1}{N_E} \sum_\alpha A_{\alpha\alpha}$$

ETH assumes $\bar{A}_{\mathrm{QM}} = \langle A \rangle_{\mathrm{QM}}$

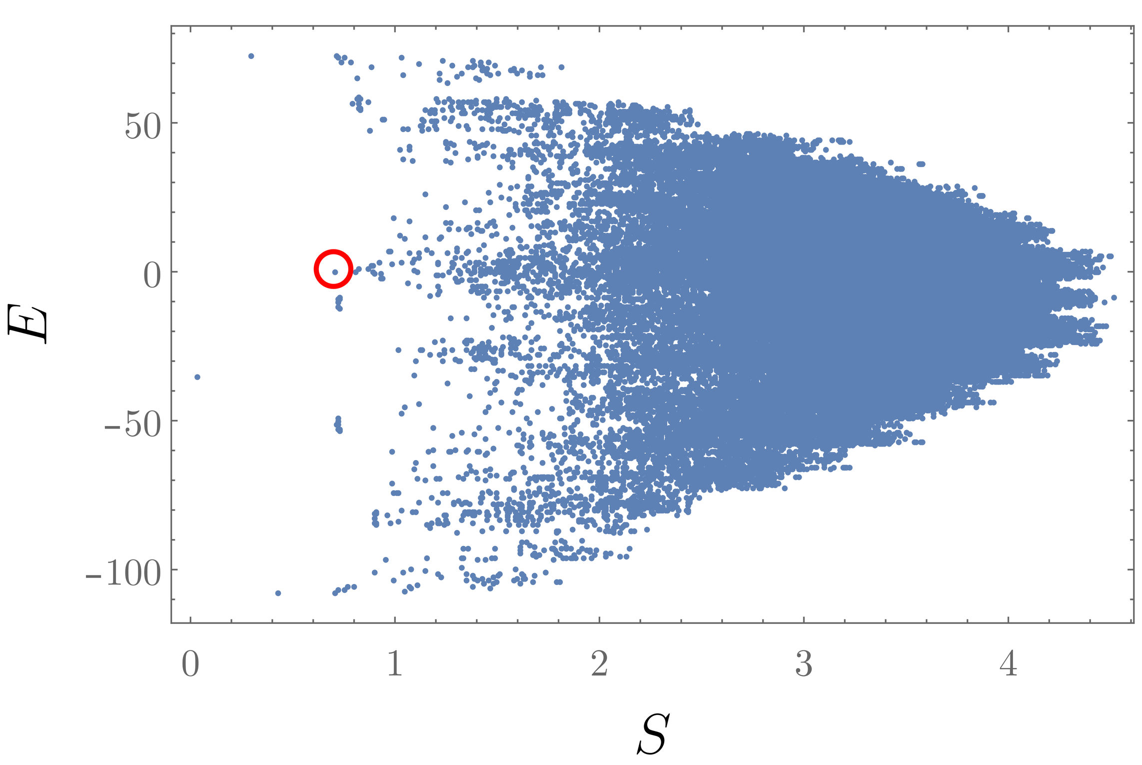
## 3. Breakdown of ETH, and thus quantum thermodynamics

- In integrable systems, where extensive number of conserved quantities exist, time evolution follows a restricted path in the phase space. The time evolution of an observable converges to the average of the expectation value only over the eigenstates in that path, at most.
- Even in non-integrable systems, where there are only a few (or completely not) conserved quantities in the level of Hamiltonian, people have found several examples in which there are special eigenstates with emergent conserved quantities (**scar states**). Those states do not obey ETH.
- Typical signal of breakdown of ETH: oscillating expectation values, sub-volume law entanglement entropy state embeded in volume law eigenstates.
- Further investigations about scar states have been restricted for the accidental nature of the scar states.
- Due to the complicity of non-integrable systems, studies have heavily relied on numerical tools.

## 4. Our construction

$$H_{\mathrm{tot}}(\beta_1,\beta_2) = \underbrace{\sum_s \left\{ \exp\left[-\beta_1 \sum_{i\in s\cap P_1} \sigma_i^z\right] - \prod_{i'\in s} \sigma_{i'}^x \right\}}_{H_1} - \underbrace{\sum_p \left\{ \exp\left[-\beta_2 \sum_{i\in p\cap P_2} \sigma_i^x\right] - \prod_{i'\in p} \sigma_{i'}^z \right\}}_{H_2}$$

- At $\beta_1=\beta_2=0$, the $H_{\mathrm{tot}}$ becomes completely solvable integrable spin models, such as spin ZXZ-chain, Kitaev's toric code, X-cube model. Tuning $\beta_1$ and $\beta_2$ away from zero switches on non-integrability.
- One can always choose $P_1$ and $P_2$ such that i) $H_1$ and $H_2$ commute each other, and ii) each of them becomes a single "macroscopic cluster" of non-integrable Hamiltonian for any $\beta_1$ and $\beta_2$.
- Each of $H_1$ and $H_2$ are in "stochastic matrix form", where the ground states are found analytically for any $\beta_1$ and $\beta_2$, respectively. Being the ground state of the both parts, the entanglement entropy scales with a logarithmic law of the boundary cut, at maximum.
- The combinations of the ground state of $H_1$ and $H_2$ are still eigenstates of $H_{\mathrm{tot}}$.
- The topological properties in $d$>1 remain untouched in the scar states.

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