The muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection

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Joint Particle Physics Seminar
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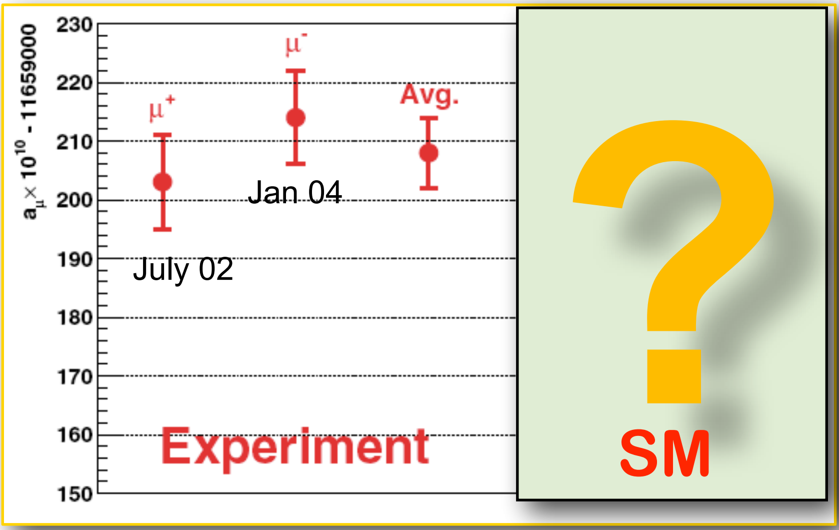
- **Muon g-2: recent theory progress**
- **Muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection**
- **The MUonE project**

Muon g-2: experimental status



- **BNL 821**: $a_\mu^{EXP} = (116592089 \pm 54_{stat} \pm 33_{sys}) \times 10^{-11}$ [0.5ppm].
- **Fermilab E989**: new muon g-2 experiment aims at $\pm 16 \times 10^{-11}$ 0.14ppm. First 3 data taking completed. Analysis of run 1 (~1xBNL) in progress. First result expected very soon with ~ BNL precision.
- **J-PARC**: Muon g-2 proposal. Phase-1 with ~ BNL precision.

Muon g-2: recent theory progress

White Paper of the Muon g-2 Theory Initiative:

arXiv:2006.04822

Muon g-2: the QED contribution

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \; 0.765857426\,(16)\,(\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$+ \; 24.05050988\,(28)\,(\alpha/\pi)^3$

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$+ \; 130.8780\,(60)\,(\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Steinhauser et al. 2013, 2015 & 2016 (all electron & τ loops, analytic);
Laporta, PLB 2017 (mass independent term). COMPLETED^2!

$+ \; 750.86\,(88)\,(\alpha/\pi)^5$ COMPLETED!

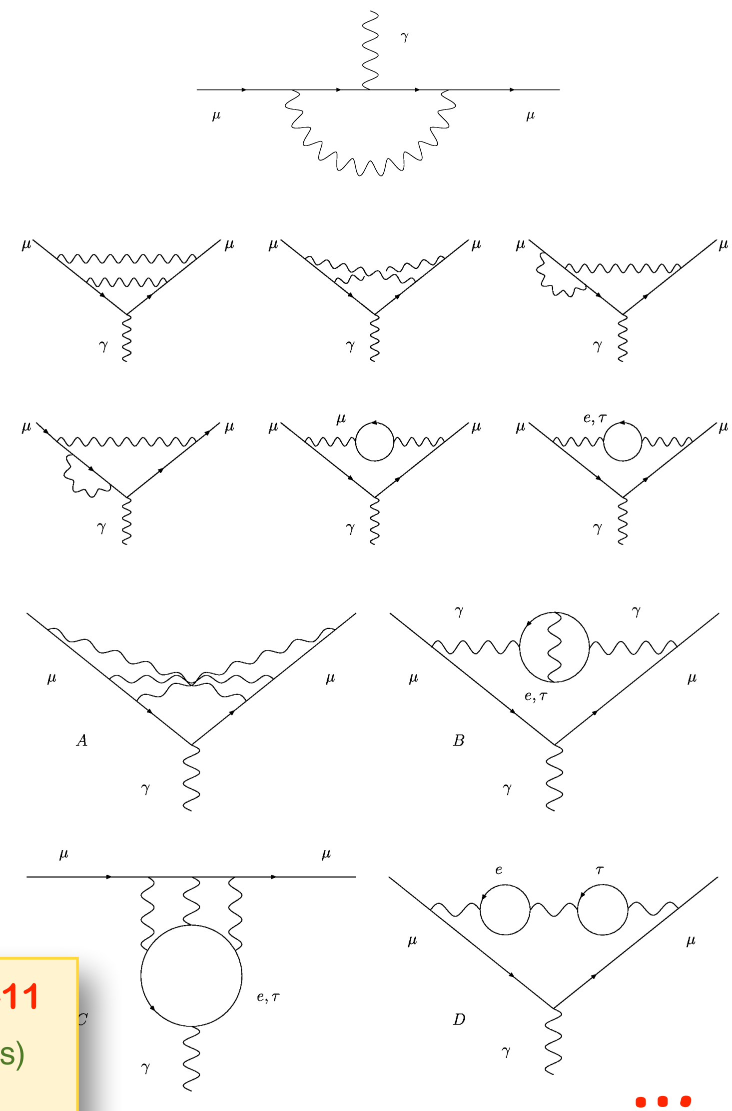
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012, 2015, 2017 & 2019.
Volkov 1909.08015: $A_1^{(10)}$[no lept loops] at variance, but negligible Δ.

Adding up, we get:

$a_\mu^{QED} = 116584718.931\,(19)(100)(23) \times 10^{-11}$

from 4-loop & 5-loop coeffs unc. 6-loop from α(Cs)

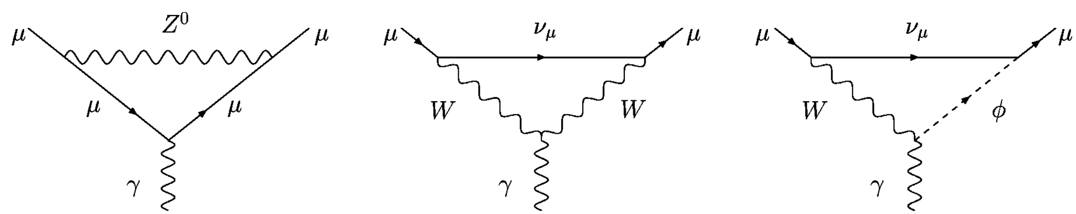
$\alpha = 1/137.035999046(27)$ [0.2ppb] 2018



Muon g-2 TI WP: arXiv:2006.04822

The electroweak contribution

- **One-loop term:**



$$a_\mu^{\rm EW}(\text{1-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

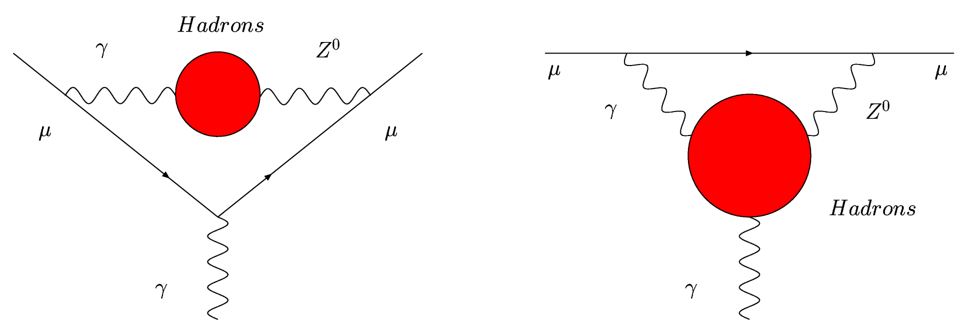
- **One-loop plus higher-order terms:**

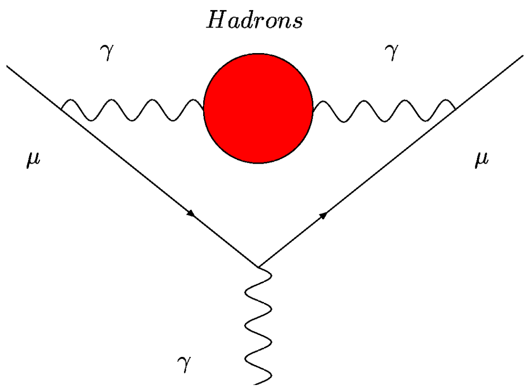
$a_\mu^{\rm EW}$ = 153.6 (1.0) x 10^{-11}

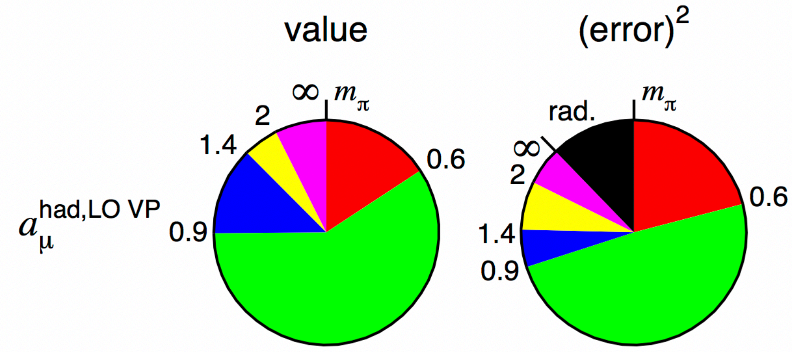
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013, Ishikawa, Nakazawa, Yasui, 2019.

Hadronic loop uncertainties (and 3-loop nonleading logs).

Muon g-2 TI WP: arXiv:2006.04822







Keshavarzi, Nomura, Teubner 2018

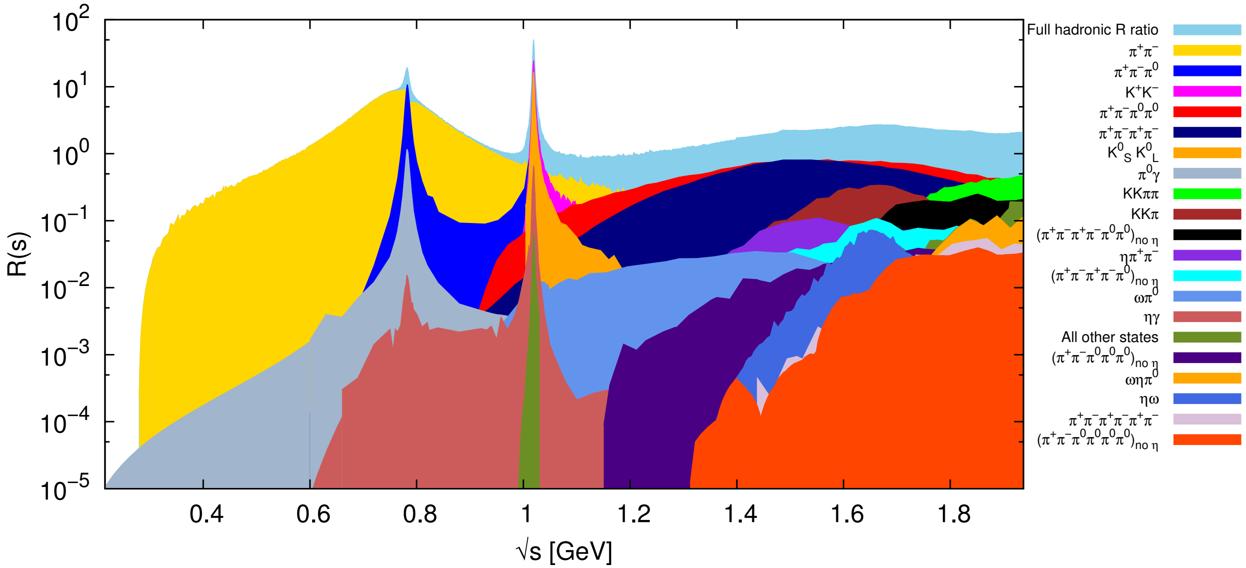
$$K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2+(1-x)(s/m^2)}$$

$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^\infty ds\, K(s)\,\sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^\infty \frac{ds}{s}\,K(s)R(s)$$

$a_\mu^{\rm HLO}$ = 6895 (33) x 10^{-11}	F. Jegerlehner, arXiv:1711.06089
= 6939 (40) x 10^{-11}	Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921
= 6928 (24) x 10^{-11}	Keshavarzi, Nomura, Teubner, arXiv:1911.00367
= 6931 (40) x 10^{-11}	Muon g-2 TI WP: arXiv:2006.04822

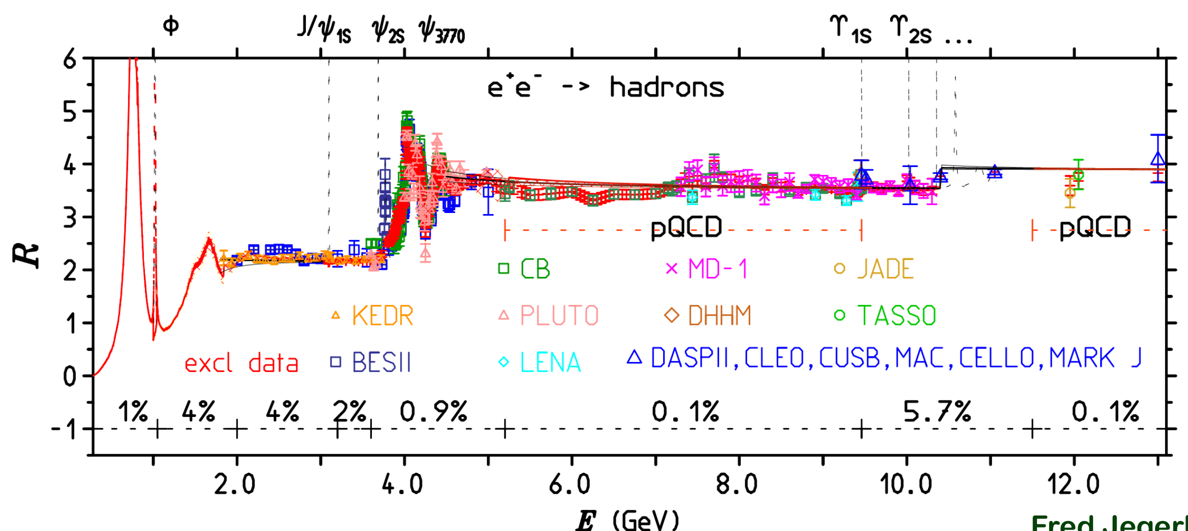
- **Radiative Corrections to σ(s) are crucial.** S. Actis et al, Eur. Phys. J. C66 (2010) 585
- **Great progress in lattice QCD results. Recent BMW result with subpercent precision: $a_\mu^{\rm HLO}$ = 7087(53)x10^{-11}. Tension with dispersive evaluations.** S. Borsanyi et al. 2002.12347.

The low-energy hadronic cross section



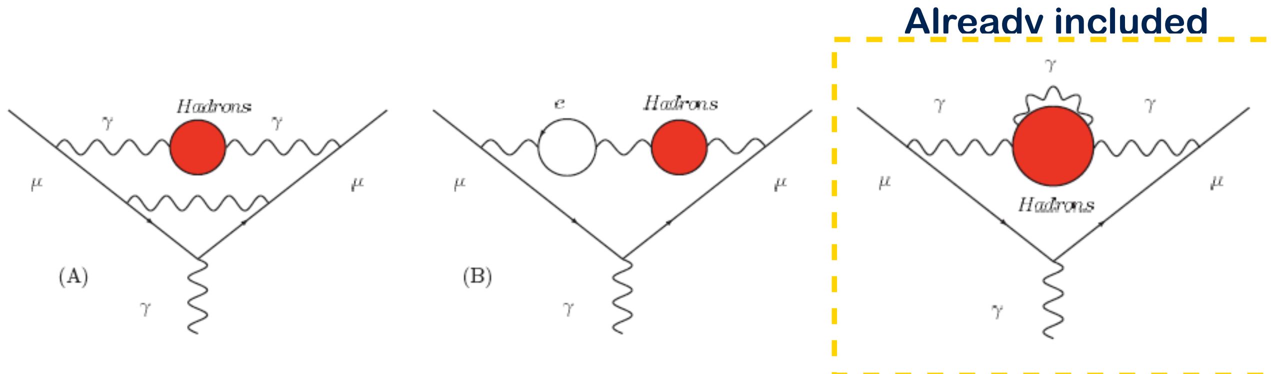
Keshavarzi, Nomura Teubner, PRD 2018

$$R(s) = \sigma(e^+e^- \to \text{hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$



Fred Jegerlehner 1905.05078

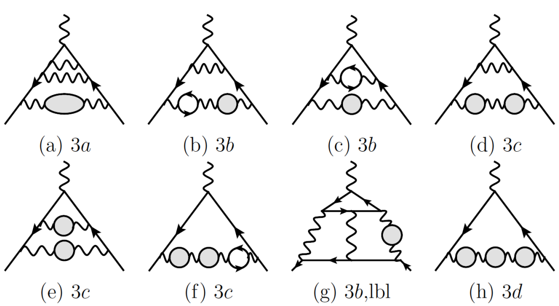
- $O(\alpha^3)$ contributions of diagrams containing HVP insertions:



$a_\mu^{HNLO}(vp) = -98.3\ (7) \times 10^{-11}$

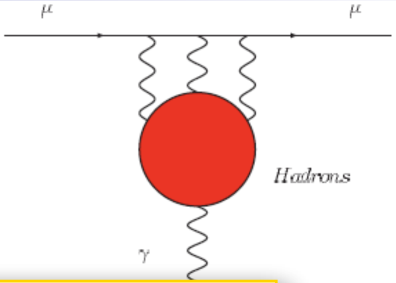
Krause '96; Keshavarzi, Nomura, Teubner 2019; Muon g-2 TI WP.

- $O(\alpha^4)$ contributions of diagrams containing HVP insertions:



$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$

Kurz, Liu, Marquard, Steinhauser 2014



HNLO light-by-light

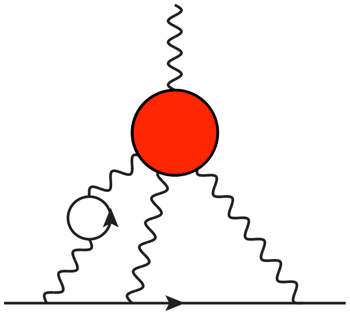
- This term had a troubled life! Nowadays:

$a_\mu^{HNLO}(lbl) =$	$+ 80\ (40) \times 10^{-11}$	Knecht & Nyffeler '02
$=$	$+136\ (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$=$	$+105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$=$	$+ 100\ (29) \times 10^{-11}$	Jegerlehner, arXiv:1705.00263
$=$	$+\ 92\ (19) \times 10^{-11}$	Muon g-2 TI WP, 2006.04822

- Significant improvements due to data-driven dispersive approach.
- Great progress on the lattice. Recent RBC result: $79(35) \times 10^{-11}$ arXiv:1911.08123

HNNLO light-by-light

$$a_\mu^{HNNLO}(lbl) = 2\ (1) \times 10^{-11}$$



Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014; Muon g-2 TI WP, 2006.04822

Comparing the SM prediction with the measured muon g-2 value:

$a_\mu^{EXP} = 116592089\ (63) \times 10^{-11}$ — BNL E821

$a_\mu^{SM} = 116591810\ (43) \times 10^{-11}$ — Muon g-2 TI

$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM} = 279\ (76) \times 10^{-11}$ — 3.7 σ

Muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection

Marciano, MP, Sirlin 2008 & 2010

Keshavarzi, Marciano, MP, Sirlin 2020

- Can Δa_μ be due to missing contributions in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$
$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\, \sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)} \rightarrow$
$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\, \sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

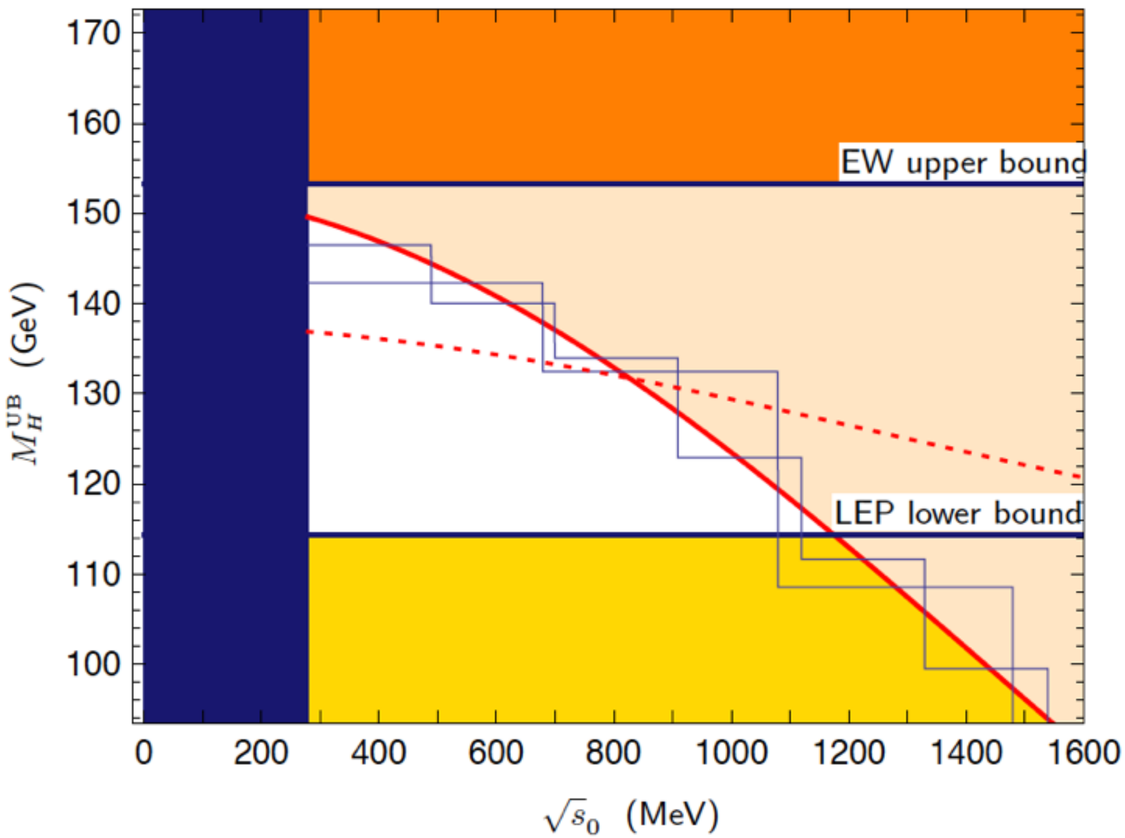
and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

ε>0, in the range:

$$\sqrt{s} \in [\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2] \longrightarrow$$

How much does the M_H upper bound from the EW fit change when we shift up $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to accommodate Δa_μ ?



Marciano, MP, Sirlin, 2008 & 2010

Muon g-2: connection with the SM Higgs mass (update)

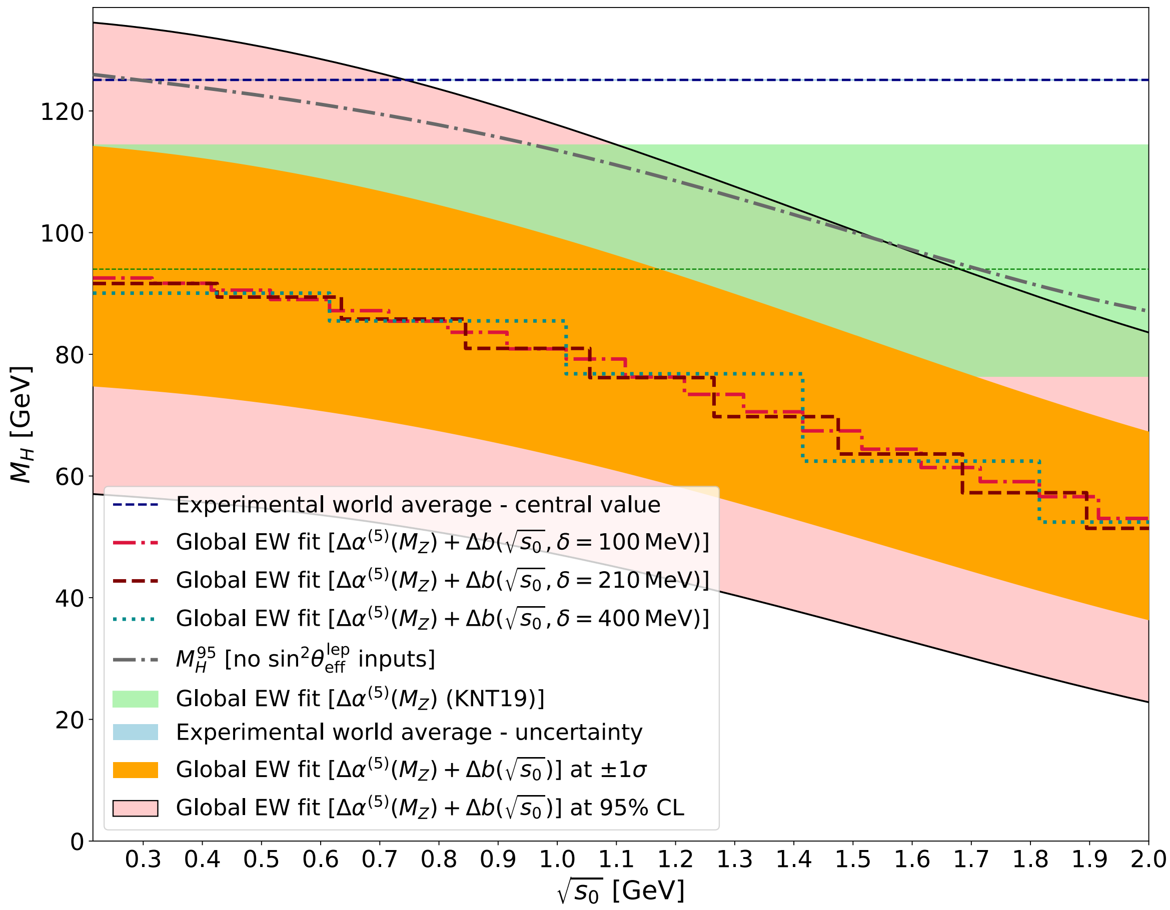
$\Delta\alpha$

Major update: Higgs discovered, improved EW observables (M_W, $\sin^2\theta$, M_{top}, …), updates to $\sigma(s)$, theory improvements, global fit, …

Parameter	Input value	Reference	Fit result	Result w/o input value
M_W (GeV)	80.379(12)	[5]	80.359(3)	80.357(4)(5)
M_H (GeV)	125.10(14)	[5]	125.10(14)	94^{+20+6}_{-18-6}
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \times 10^4$	276.1(1.1)	[23]	275.8(1.1)	272.2(3.9)(1.2)
m_t (GeV)	172.9(4)	[5]	173.0(4)	...
$\alpha_s(M_Z^2)$	0.1179(10)	[5]	0.1180(7)	...
M_Z (GeV)	91.1876(21)	[5]	91.1883(20)	...
Γ_Z (GeV)	2.4952(23)	[5]	2.4940(4)	...
Γ_W (GeV)	2.085(42)	[5]	2.0903(4)	...
$\sigma^0_{\rm had}$ (nb)	41.541(37)	[108]	41.490(4)	...
R^0_l	20.767(25)	[108]	20.732(4)	...
R^0_c	0.1721(30)	[108]	0.17222(8)	...
R^0_b	0.21629(66)	[108]	0.21581(8)	...
$\bar{m}_c$ (GeV)	1.27(2)	[5]	1.27(2)	...
$\bar{m}_b$ (GeV)	$4.18^{+0.03}_{-0.02}$	[5]	$4.18^{+0.03}_{-0.02}$	...
$A^{0,l}_{\rm FB}$	0.0171(10)	[108]	0.01622(7)	...
$A^{0,c}_{\rm FB}$	0.0707(35)	[108]	0.0737(2)	...
$A^{0,b}_{\rm FB}$	0.0992(16)	[108]	0.1031(2)	...
A_ℓ	0.1499(18)	[75,108]	0.1471(3)	...
A_c	0.670(27)	[108]	0.6679(2)	...
A_b	0.923(20)	[108]	0.93462(7)	...
$\sin^2\theta^{\rm lep}_{\rm eff}(Q_{\rm FB})$	0.2324(12)	[108]	0.23152(4)	0.23152(4)(4)
$\sin^2\theta^{\rm lep}_{\rm eff}$(Had Coll)	0.23140(23)	[100]	0.23152(4)	0.23152(4)(4)

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (using Gfitter)

Muon g-2: connection with the SM Higgs mass (2020)

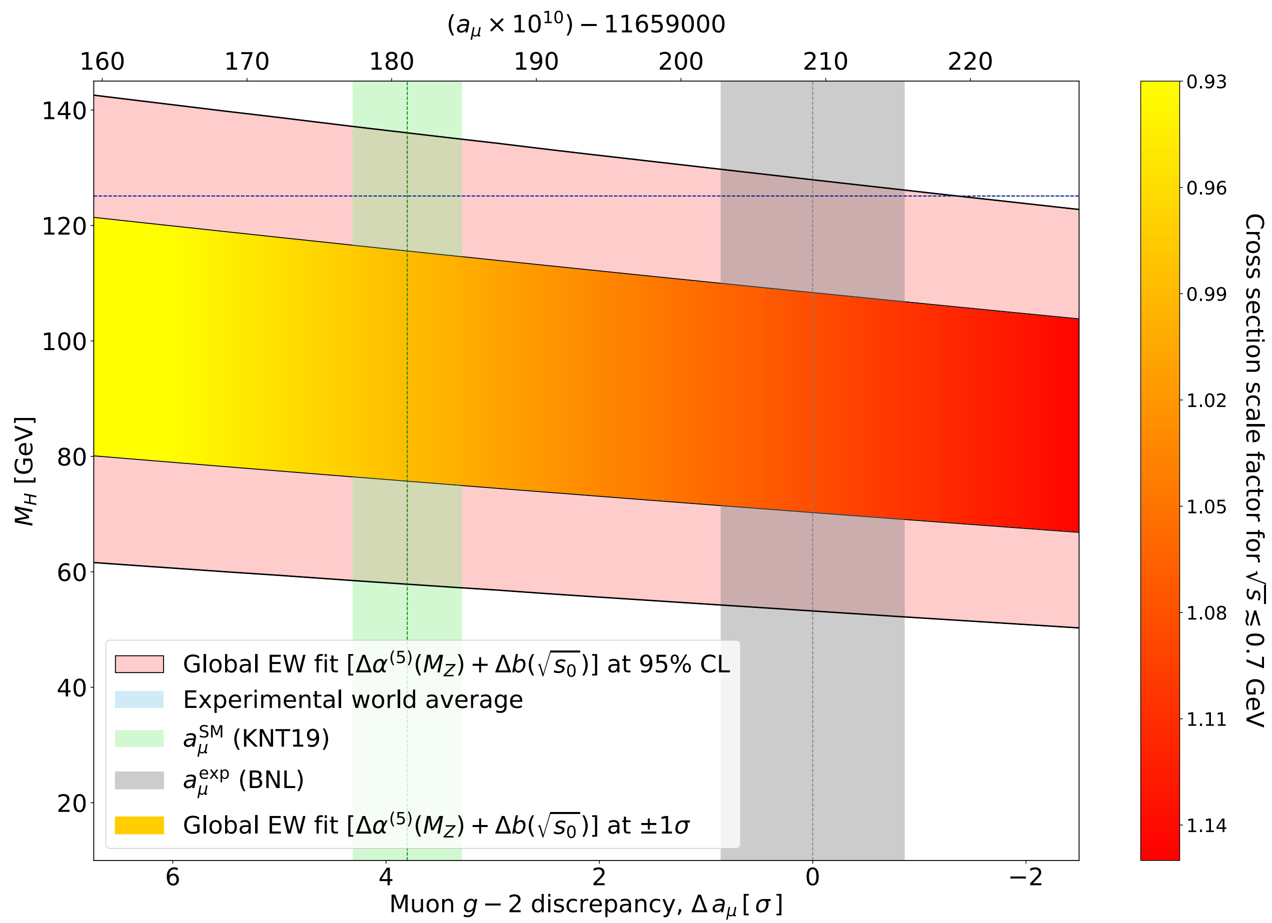


Shifts $\Delta\sigma(s)$ to fix Δa_μ are possible, but conflict with the EW fit if they occur above ~1 GeV

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

The muon g-2: connection with the SM Higgs mass (2020) - II

Δα

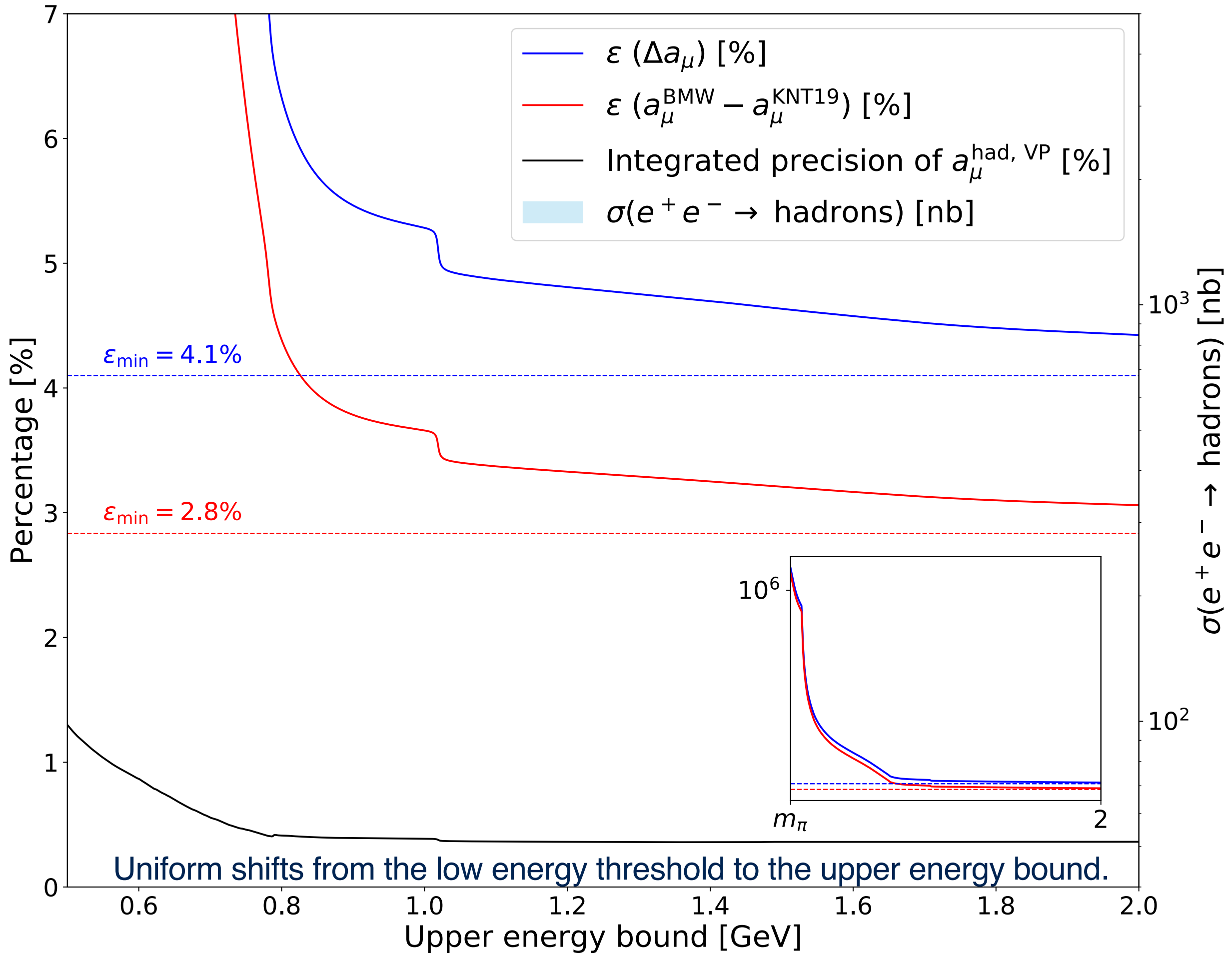


Uniform scaling of σ(s) below ~0.7 GeV? +9% required!

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

How large are the required shifts $\Delta\sigma(s)$?

$\Delta\alpha$



Shifts below 1 GeV conflict with the quoted exp. precision of $\sigma(s)$

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

What happens to the electron g-2?

The electron g-2 no longer provides the best value of α

- The 2008 measurement of the electron g-2 is:

  $a_e^{EXP} = 11596521807.3\ (2.8) \times 10^{-13}$ Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):

  $a_e^{EXP} = 11596521883\ (42) \times 10^{-13}$ Van Dyck et al, PRL59 (1987) 26

- Equate $a_e^{SM}(\alpha) = a_e^{EXP}$ → "g_e-2" determination of alpha:

$$\alpha^{-1} = 137.035\,999\,151\ (33) \quad [0.24\ \text{ppb}]$$

- The best determination of α is obtained via atomic interferometry:

$\alpha^{-1} = 137.036\,999\,046\ (27)$ [0.20 ppb] Science 360 (2018) 191 (Cs)

(was α^{-1} =137.035 998 995 (85) [0.62 ppb] PRL106 (2011) & CODATA 2016)

2.5 sigma discrepancy

Electron g-2: SM vs experiment

e

- Using $\alpha = 1/137.036\,999\,046\,(27)$ [Cs 2018], the SM prediction for the electron g-2 is:

$$a_e^{SM} = 115\,965\,218\,16.2\,(0.1)\,(0.1)\,(2.3) \times 10^{-13}$$

δC_5^{qed} δa_e^{had} from $\delta\alpha$

- The (EXP – SM) difference is:

NB: negative!

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -\,8.9\,(3.6) \times 10^{-13}\ [2.5\sigma]$$

Note the negative sign. [QED 5-loop $a_e^{QED5} = 4.6 \times 10^{-13}$]

- NP sensitivity limited only by the experimental errors in α and a_e. May soon play a pivotal role in probing NP in the leptonic sector.

Giudice, Paradisi, MP 2012

- The present sensitivity is $\delta\Delta a_e = 3.6 \times 10^{-13}$, ie ($10^{-13}$ units):

$$\underbrace{(0.1)_{\rm QED5}, \quad (0.1)_{\rm HAD}}_{(0.1)_{\rm TH}}, \quad (2.3)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\rm EXP}}$$

- The $(g\text{-}2)_e$ exp. error may soon drop below 10^{-13} and work is in progress to further reduce the error induced by $\delta\alpha$ →

sensitivity below 10^{-13} may be reached with ongoing exp work

- In a broad class of BSM theories, contributions to a_l scale as

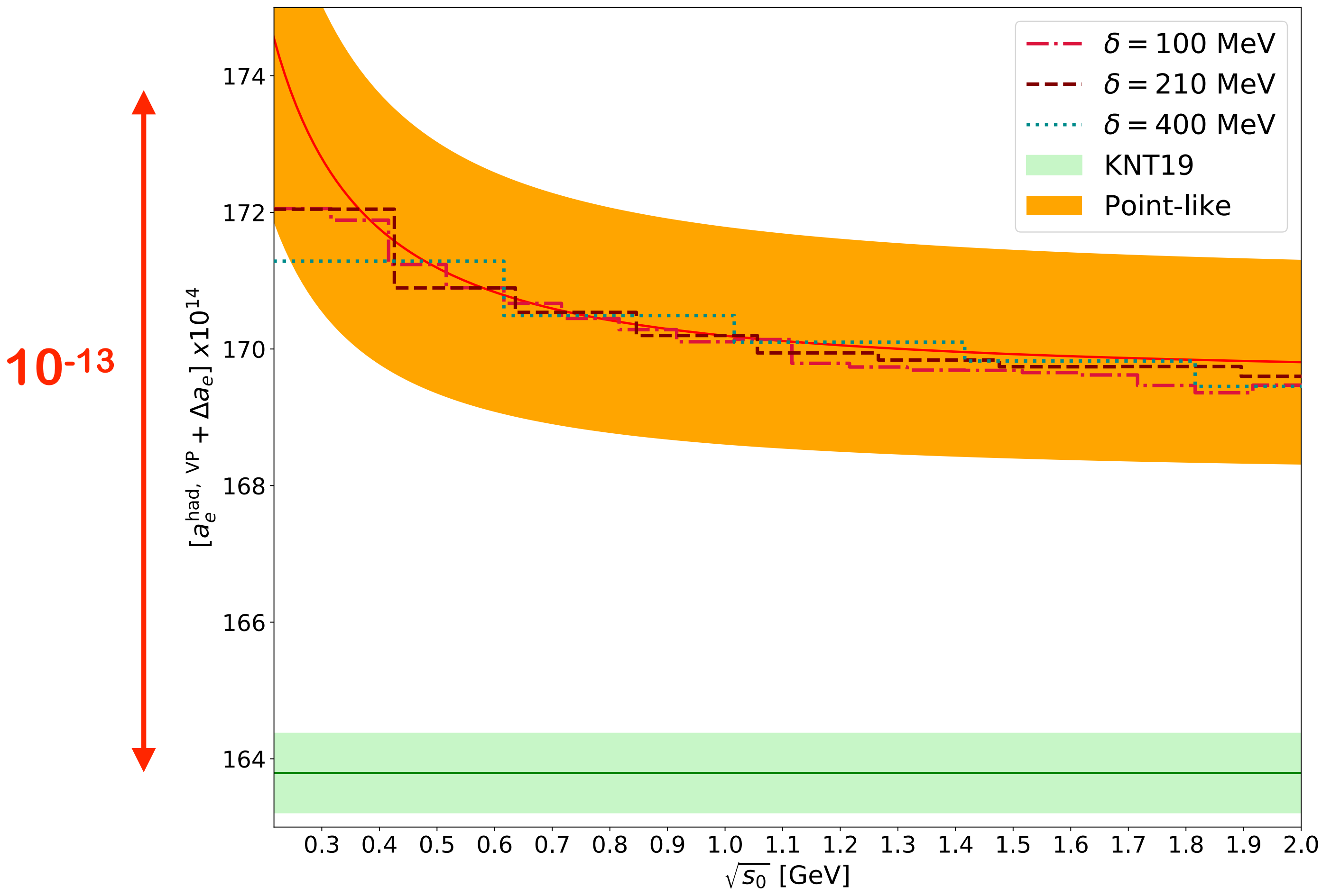
$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This **Naive Scaling** leads to:

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}; \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}$$

Giudice, Paradisi & MP, JHEP 2012

Shift of the electron g-2

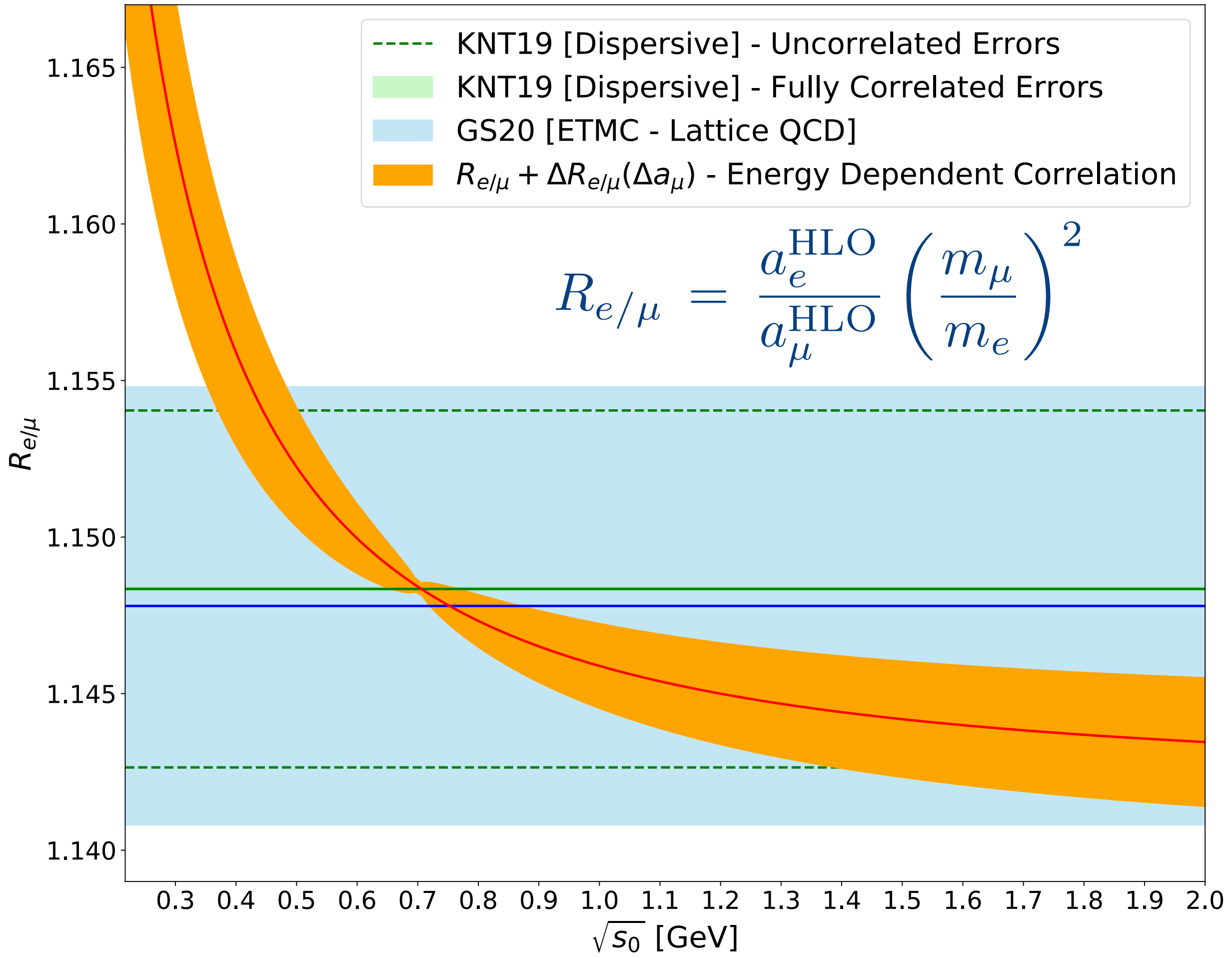


Shifts $\Delta\sigma(s)$ to fix Δa_μ slightly increase the $|\Delta a_e| \sim 10^{-12}$ tension

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

Shift of the e/μ g-2 scaled HLO ratio

e/μ



Good agreement between lattice [Giusti & Simula 2020] and KNT19.
Possible future bounds on very low energy shifts Δσ(s)?

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

$a_\mu \Longleftrightarrow \Delta\alpha$: recent related literature

- **Crivellin, Hoferichter, Manzari and Montull, "Hadronic vacuum polarization: $(g\text{-}2)_\mu$ versus global electroweak fits,'' PRL125 (2020) 091801 [arXiv:2003.04886].**

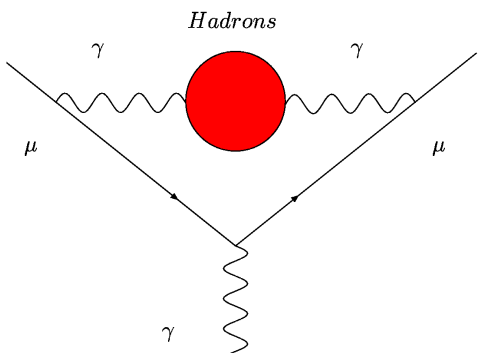
- **Eduardo de Rafael, "On Constraints Between $\Delta\alpha_{had}(M_Z^2)$ and $(g_\mu\text{-}2)_{HVP}$,'' arXiv:2006.13880.**

- **Malaescu and Schott, "Impact of correlations between a_μ and α_{QED} on the EW fit", arXiv:2008.08107.**

- **Colangelo, Hoferichter and Stoffer, "Constraints on the two-pion contribution to hadronic vacuum polarization," arXiv:2010.07943.**

The MUonE project

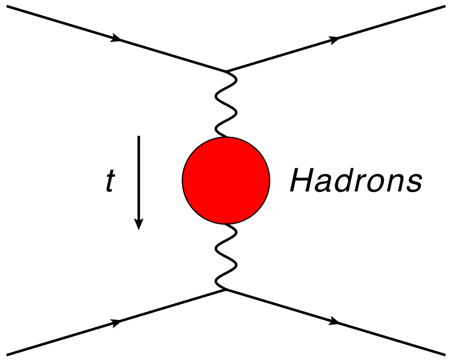
- The leading hadronic contribution a_μ^{HLO} computed via the timelike formula:



$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{\rm had}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2\,(1-x)}{x^2 + (1-x)\,\left(s/m_\mu^2\right)}$$

- Alternatively, simply exchanging the x and s integrations:



$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

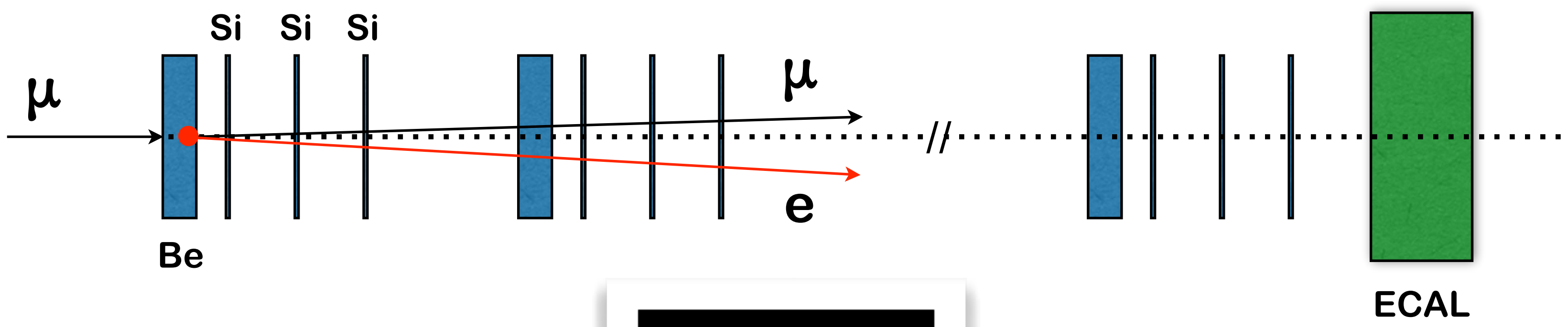
$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the spacelike region: a_μ^{HLO} can be extracted from scattering data!

Carloni Calame, MP, Trentadue, Venanzoni, 2015

MUonE: Muon-electron scattering @ CERN

- $\Delta\alpha_{had}(t)$ can be measured via the elastic scattering $\mu\, e \to \mu\, e$.

- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.



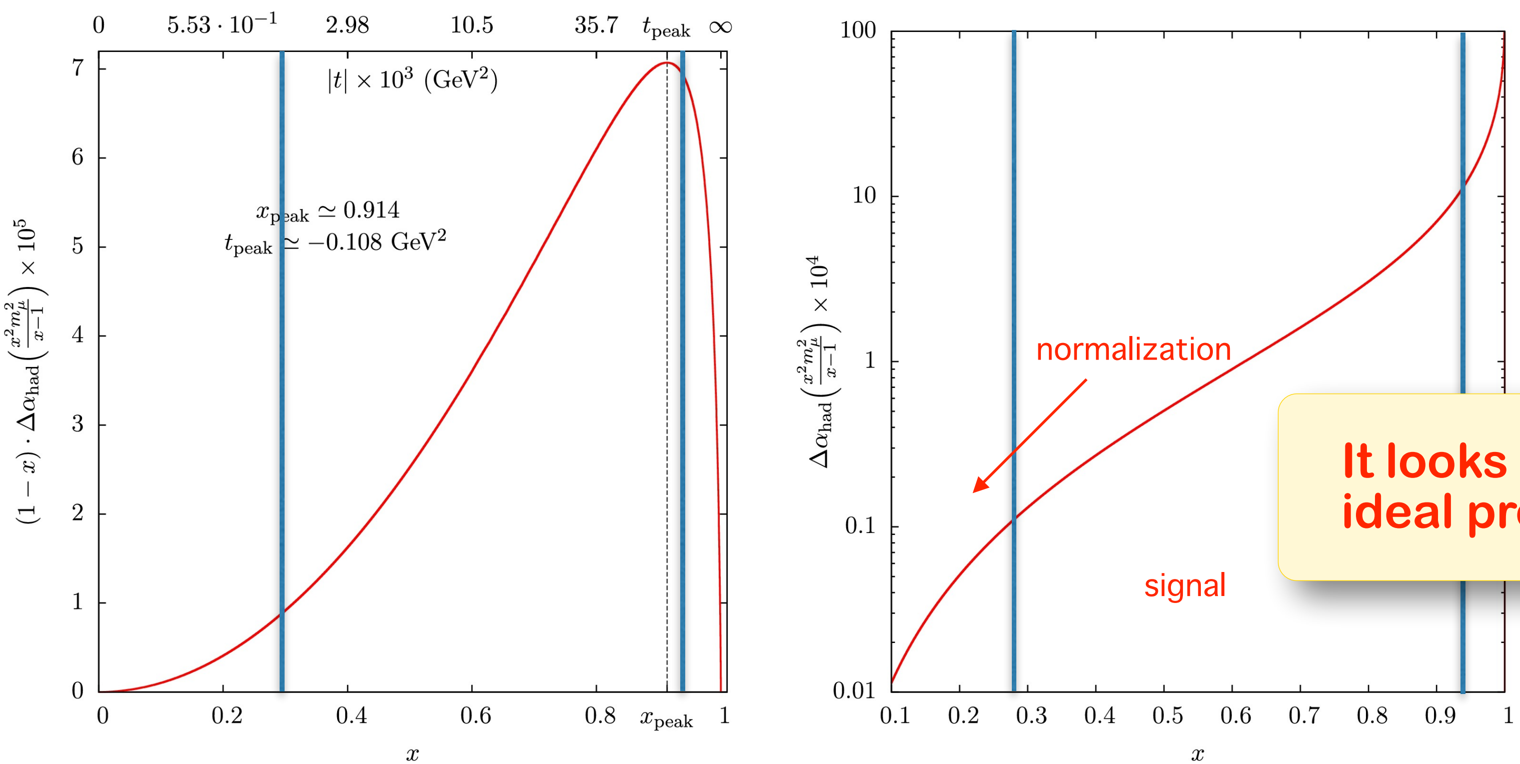
Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna,

Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni

EPJC 2017 - arXiv:1609.08987

MUonE: Muon-electron scattering @ CERN (II)

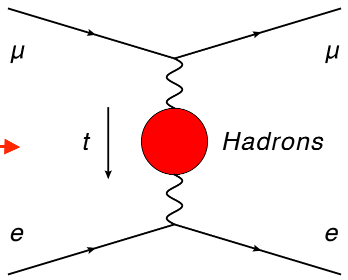
- For a 150 GeV muon beam, MUonE's scan region extends up to x=0.932, ie beyond the peak! (the peak is at x=0.914)
- The high-energy region inaccessible to MUonE contributes only 13% of a_μ^{HLO} integral. It can be determined with timelike data and/or lattice QCD results. Recently obtained via lattice QCD. Giusti&Simula and Marinkovic´&Cardoso 2019



It looks like an ideal process!

- Statistics: With CERN's 150 GeV muon beam M2 (1.3×10^7 μ/s), incident on 40 15mm Be targets (total thickness 60cm), 2-3 years of data taking ($2{\times}10^7$ s/yr) → $\mathscr{L}_{int} \sim 1.5 \times 10^7$ nb^{-1}.
- With this $\mathscr{L}_{int}$ we estimate that measuring the shape of dσ/dt we can reach a statistical sensitivity of ~0.3% on a_μ^{HLO}, ie $\sim 20 \times 10^{-11}$.
- Systematic effects must be known at ≲ 10ppm!
- Test beams performed at CERN in 2017 & 2019 arXiv:1905.11677
- LoI submitted to CERN SPSC in 2019: Test run approved for 2021.
- Full-statistics run hopefully in 2022–24.

- To extract $\Delta\alpha_{had}(t)$ from MUonE's measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at ≲ 10ppm!



- Fully differential fixed-order MC @ NLO ready Pavia and PSI 2018-19
- NNLO QED: MI for 2-loop box diagrams computed Padova 2017-19
- Two MC built including partial subsets of the NNLO QED corrections due to electron and muon radiation Pavia and PSI 2020
- NNLO hadronic effects computed Padova and Siegen 2019
- Extraction of the leading electron mass effects from the massless muon-electron scattering amplitudes PSI 2019-2020
- New physics at MUonE Heidelberg&Washington and Padova 2020
- …

Theory for muon-electron scattering @ 10 ppm:
A report of the MUonE theory initiative. arXiv:2004.13663

2^{nd} Workstop / Thinkstart: 4^{th} – 7^{th} Feb 2019

Theory for muon-electron scattering @ 10ppm

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Organized by
A. Signer & Y. Ulrich

MUonE theory workshops: Padova 2017, Mainz 2018, Zurich 2019

Next MUonE theory workshop: MITP Mainz 01.03 - 05.03 2021 (hopefully)

Conclusions

- FNAL's first a_μ result expected very soon with ~BNL precision.
- Is present Δa_μ due to missed contributions in the hadronic $\sigma(s)$?

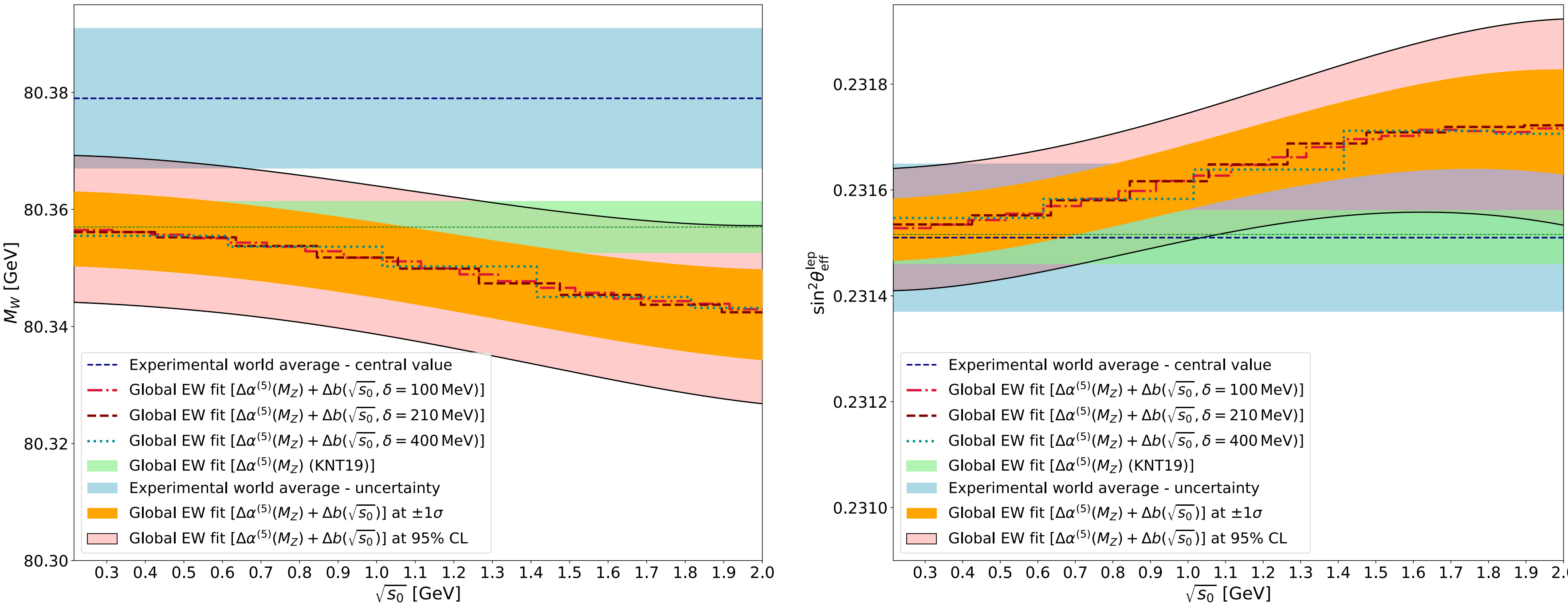
  Shifts $\Delta\sigma(s)$ to fix Δa_μ conflict with the global EW fit above ~1 GeV
  Shifts below ~1 GeV conflict with the quoted exp. error of $\sigma(s)$.

- Shifts $\Delta\sigma(s)$ to fix Δa_μ slightly increase the a_e tension ($R_{e/\mu}$ ok).
- MUonE at CERN will provide a new independent spacelike determination of a_μ^{HLO} alternative to the DR and lattice ones.

Backup

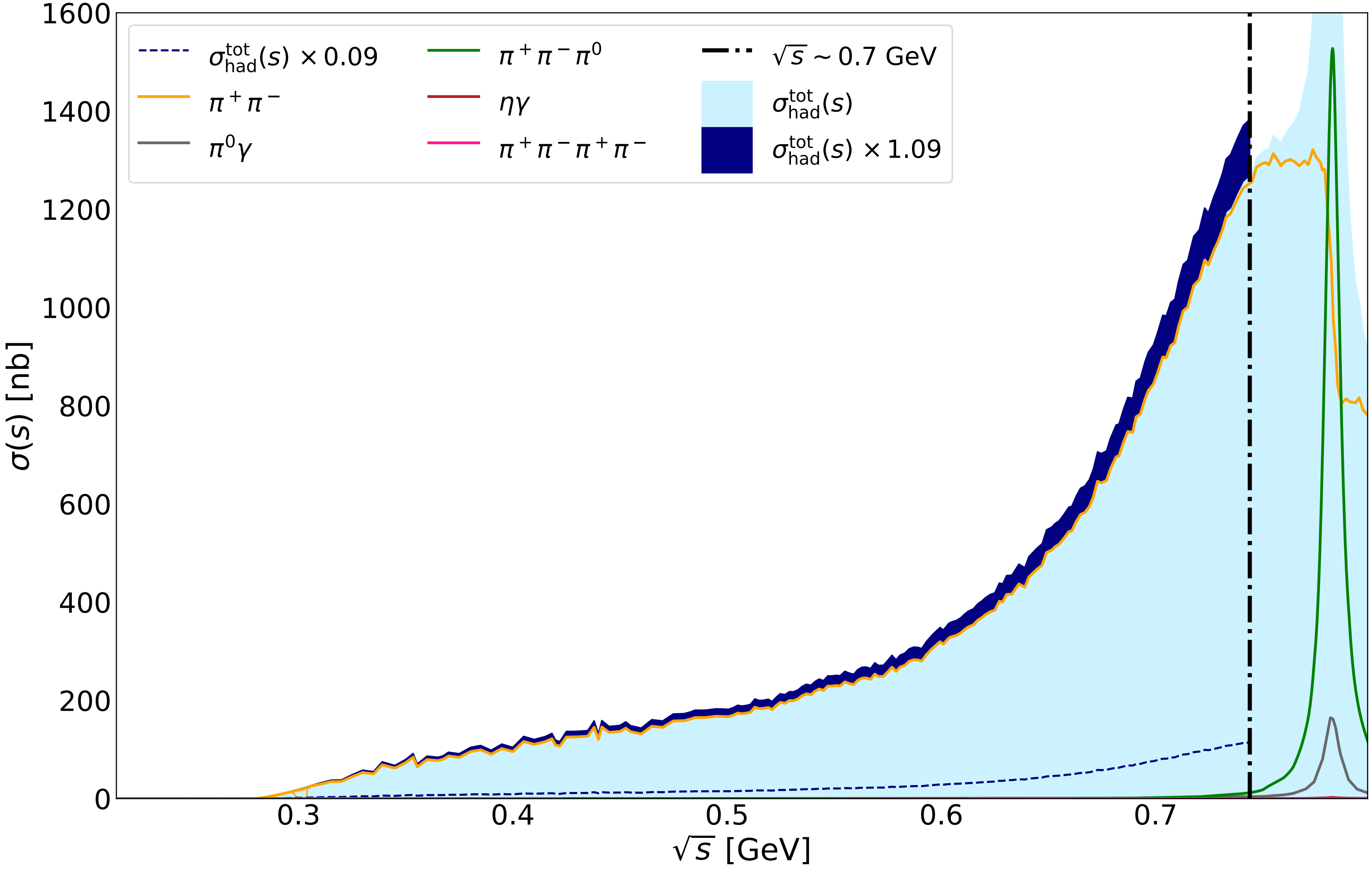
Muon g-2: connection with M_W and $\sin^2\theta$

Δα

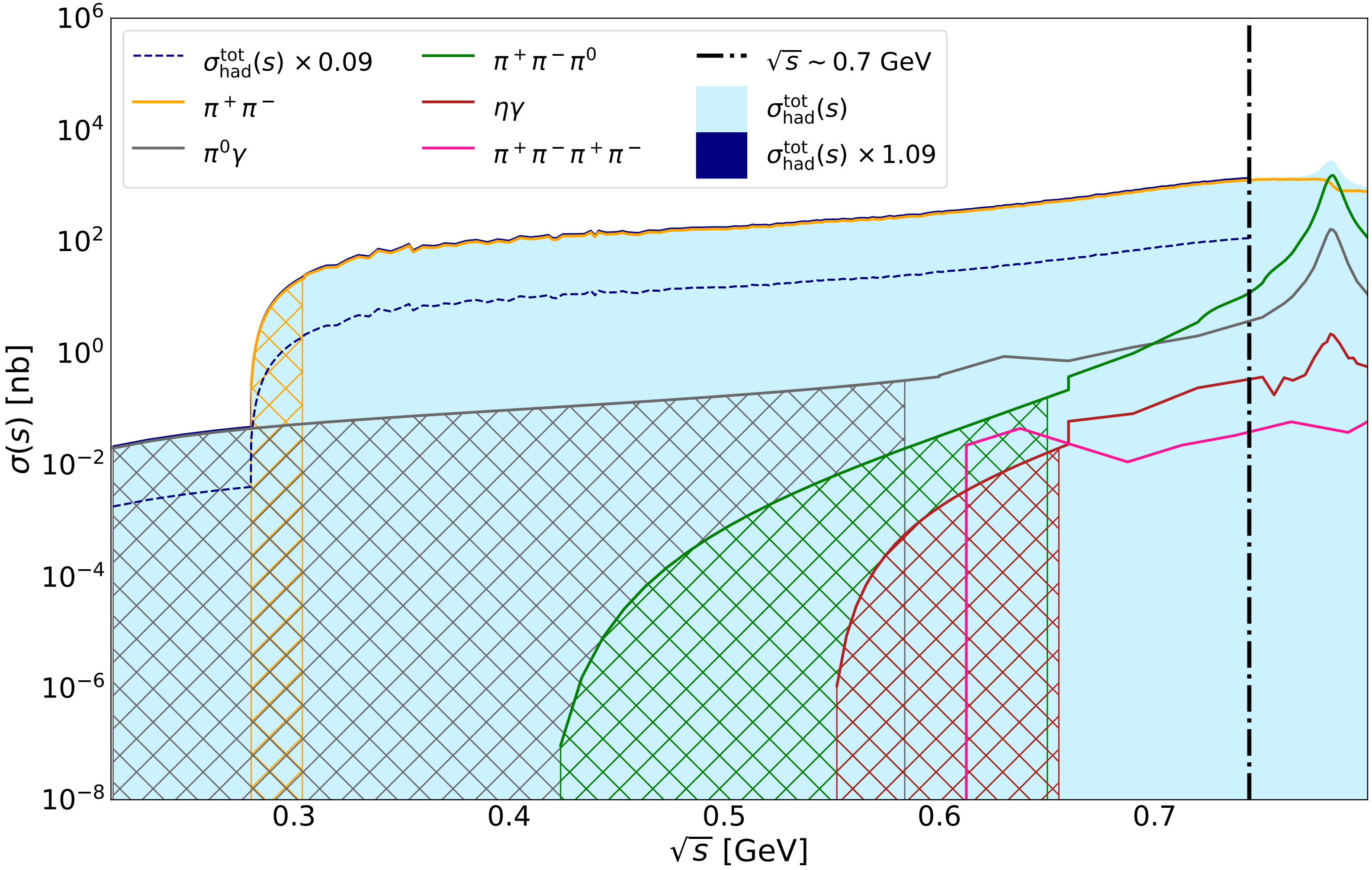


Keshavarzi, Marciano, MP, Sirlin, PRD 2020

Δα



Keshavarzi, Marciano, MP, Sirlin, PRD 2020



Keshavarzi, Marciano, MP, Sirlin, PRD 2020